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New Product Launch Decisions under Competition and Uncertainty: A Real Options and Game-Theoretic Approach to New Product Development

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NEW PRODUCT LAUNCH DECISIONS UNDER COMPETITION AND UNCERTAIANTY: A REAL OPTIONS AND GAME-THEORETIC APPROACH TO NEW PRODUCT DEVELOPMENT

by

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ABSTRACT

NEW PRODUCT LAUNCH DECISIONS UNDER COMPETITION AND UNCERTAIANTY: A REAL OPTIONS AND GAME-THEORETIC APPROACH TO NEW PRODUCT DEVELOPMENT

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School of Technology

Master of Science

New product development is central to many firms' future success. Not only as a means to continue to maintain their piece of the market, but product development can also be a strategic means for a company to diversify, and/or alter focus to adapt to changing market conditions.

Most of the research in new product development has been on how to do it cheaper and faster than the next guy. However, early commercialization does not guarantee a position of strength in the market. Failures of EMI in CT scanners and Xerox in personal computers illustrate that being first to market does not ensure success or even survival. There are two main factors that inhibit managers from making educated

decisions on when to introduce a new product. First, firms do not exist in a vacuum and any action they take will be countered by their competition. Second, with new products the only certainty is uncertainty.

To allow such decisions to become “gut feeling” decisions puts a company’s future at unnecessary risk. This is evidenced by the many firms that have had devastating results because of poor decisions with regard to launching a new product.

While high level quantitative tools have recently begun to be used to evaluate corporate strategy, these tools are still mainly confined to research groups within large corporations. Both real options (to handle uncertainty) and game theory (to capture the effects of the competitions actions) have been evaluated and used by these groups. However, they have not been adequately integrated together in the academic world, let alone in industry. This thesis help bridge the gap between strategic decision making, and the theoretical world of economic decision analysis creating a prescriptive model companies can use to evaluate strategically important new product launches.

To bridge this gap a method that is able to handle the integration of game-theoretic and options-theoretic reasoning to the strategic analysis of new product introduction is developed. Not only was a method developed that could incorporate the two methods it was done in a way that is accessible and useful outside of the academic world.

ACKNOWLEDGMENTS

I would like to thank my committee for their support in my efforts to accomplish my goals. I would also like to give special thanks to my wife Megan who has supported me in everything that I do.

Table of Contents

CHAPTER 1. THESIS MOTIVATION AND FRAMEWORK 1

1.1 Model 2

1.2 Methodology 5

1.3 Problem Statement 7

1.4 Delimitations 9

CHAPTER 2. LITERATURE REVIEW 11

2.1 Importance of New Product Development 11

2.2 Why Develop Products Faster 12

2.2.1 Quick product development time 12

2.2.2 First mover advantage 13

2.3 Costs of Speed 16

2.3.1 Time cost tradeoff 16

2.3.2 Competing objectives. 17

2.4 Time-to-Market Tradeoff 20

2.4.1 First-mover disadvantages 20

2.4.2 Market timing 20

2.5 New Product Evaluation 21

2.5.1 Discounted cash flow 21

2.5.2 Real options 22

2.5.3 Options and new product development 24

2.6 Competitive Response and Game Theory 25

2.6.1 Game theory 25

2.6.2 Timing of technology introduction in a duolpoly 30

2.6.3 Integrating real options and game theory 30

CHAPTER 3. METHODOLGY FOR MODELING THE INVESTMENT DECISION . 33

3.1 Game-Theoretic Analysis 34

3.2 Real Options Analysis 39

3.3 Cash Flow 42

3.4 The Demand Model 45

3.5 Assumptions 51

3.6 Model Programming 52

CHAPTER 4. METHODOLGY OF ANALYSIS 53

4.1 Monte Carlo Simulation 53

4.2 The Hazard Model 55

CHAPTER 5. ANALYSIS AND RESULTS 59

5.1 Using the Hazard Model to Make Real Time Decisions 59

5.2 Real Time Decision Making 60

5.2.1 Testing hypothesis 61

5.2.2 Risk management 65

5.2.3 Crosstabulation 65

CHAPTER 6. CONCLUSIONS AND FUTURE RESEARCH 69

6.1 Conclusions 69

6.2 Future Research.. 70

REFERENCES 71

APPENDIX.. 77

List of Tables

Table 3-1 – Normal form entry game with a 20 year horizon 35

Table 3-2 – Assumed parameters for simulation models 51

Table 4-1 – Variable matrix 54

Table 5-1 – Boeing survival table 63

Table 5-2 – Variables in the equation 64

Table 5-3 – Airbus survival table 64

Table 5-4 –Variables in the equation 65

Table 5-5 – BL*DO*CIA Crosstabulation 66

List of Figures

Figure 1-1 – A 2x2 matrix of NPV payoffs 5

Figure 2-1 – Endogenous generation of first-mover advantages 14

Figure 2-2 – The development time cost tradeoff.. 17

Figure 2-3 – Four key product development objectives and six tradeoffs........................ 18

Figure 2-4 –Thee representative forms of a game 29

CHAPTER 1. THESIS MOTIVATION AND FRAMEWORK

New product development is central to many firms' future success. Not only as a means to continue to maintain their piece of the market, but product development can also be a strategic means for a company to diversify, and/or alter focus to adapt to changing market conditions (Schoonhoven, Eisenhardt, & Lyman, 1990).

Most of the research in new product development has been on how to do it cheaper and faster than the next guy. Manufacturing was usually pushed to find new ways of producing products faster in the goals of being first to market. However, this world is not so simple. Early commercialization does not guarantee a position of strength in the market. Failures of EMI in CT scanners and Xerox in personal computers illustrate that being first to market does not ensure success or even survival (Teece 1986). Recent work by Lieberman and Montgomery (1988, 1998) shows that late movers can enjoy advantages such as: (1) free-riding on the first mover's investments, (2) technological and market uncertainty, (3) technological discontinuities, (4) incumbent inertia of the first mover making it difficult to adapt to change.

A new product's success also depends on its timing. Abell (1978) introduced the concept of a "strategic window of opportunity". Entry which is too late represents lost opportunity; on the other hand, a product introduced to the market too early may not be received by customers or market channels.

There are two main factors that inhibit managers from making educated decisions on when to introduce a new product. First, firms do not exist in a vacuum and can be assured that any action they take will be seen and countered by their competition. Second, the only certainty in the world of new products is uncertainty.

To allow such decisions to become “gut feeling” decisions puts a company’s future at unnecessary risk. This is evidenced by the many firms that have had devastating results because of poor decisions with regard to launching a new product. There are tools that deal with each of these factors separately. Game theory can help understand competitors expected response, and real options can deal with the uncertainty of the market. However, neither of these tools alone will incorporate all of the information necessary to make an educated decision.

This research joins with the recent work of Smit and Trigeorgis (2001), Smit and Ankum (1993), and Kulatilaka and Perotti (1998) to add the influence of rivals into real options analysis of strategic investment and explicitly introduce the resulting tension between the value of commitment and the value of flexibility in the introduction of new products.

1.1 Model

For this study the chosen context to explore the dynamics of entry timing choices is in the market for “very large aircraft” (VLA) aircraft. Boeing has held an unchallenged monopoly in the VLA aircraft market for almost 40 years with the 747 aircraft. Boeing’s monopoly in the VLA market plays a critical role for the company in two ways: first, Boeing earns substantial monopolistic profits on their pricing of the 747; second, Boeing uses these monopolistic profits to subsidize other plane segments where they compete against Airbus. Recognizing the profit potential from breaking Boeing's monopoly, Airbus has repeatedly announced its intention to build the A380, a larger VLA than any previously built. Airbus's motivation for entering the market is both to tap the profits in the VLA segment as well as the competitive advantage of the VLA monopoly position that Boeing has been enjoying and leveraging.

Boeing's daring gamble in 1965 launching the 747 jumbo jet was one of the main reasons that they are the industry leader in the oligopoly market of aerospace manufacturers, and until Airbus began looking at developing the A380 they enjoyed monopolistic returns being the sole provider of an aircraft in the super-jumbo jet category. Now the potential entry of a larger and more efficient aircraft than the 747 Boeing is faced with a critical strategic decision of how to respond to this new entry. If they do nothing they may lose their position in the aerospace market along with billions of dollars. On the other hand, launching their own new jumbo jet may not be the answer since recently failed launches by companies such as the Glenn Martin Company and Lockheed, have proven devastating. Even the successful launch of the 747 had almost failed, and Boeing cannot afford to make a mistake that will cost them billions of dollars.

However, entry by Airbus into the superjumbo segment would do considerable damage to Boeing. The increased size and efficiency of the A380 would likely put significant competitive pressure on Boeing either reducing margins on the existing 747 or forcing Boeing to launch a new plane. While Boeing did not have plans to launch a completely new aircraft, it was believed that a revised 747 (with increased efficiency and seating) could be launched for approximately $2-3 billion. Boeing is concerned with how the competitive response to either launching or not launching will change what airbus is planning to do. For example, Boeing needs to know whether the launch of this new plane would effectively blockade Airbus' entry.

On the other hand, the decision facing Airbus is very important and complex. First, the market for intercontinental jumbo jets is predicted to experience significant growth over the coming decades as traffic on Pacific routes expanded. In addition, Airbus is under increasing pressures from its customers to provide a full line of aircraft. However, at the same time, Airbus faces significant risk. The capital investment required for the project is sizeable; if demand for the plane fails to materialize, the financial viability of the company could be endangered.

By building a model of the situation, possible outcomes can be explored and decisions rules found. While the future cannot be predicted perfectly by a model, different decision policies can be evaluated and then used in the actual decisions that need to be made. Two economic approaches are used to help understand and breakdown the problem, game-theory and real options. Both of these approaches can be applied to the Bertrand Oligopoly situation that arises in the airline industry.

While game theory focuses on the effects resulting from strategic interaction, real options concerns itself with decision-making under uncertainty. In particular, real options theory is concerned with decisions where current decisions have implications for future investment opportunities. In this case the airlines are presented with a buy (launch) or wait scenario. By waiting for more information about future conditions they may increase their expected return on investment.

1.2 Methodology

In the Boeing-Airbus case the stochastic nature of the demand can be modeled by a Markov Chain (Benkard 2000). This assumption allows for the model to solve for the dynamic possibilities instead of stationary situations. However, the payoff that the airlines will receive is dependent upon more that just the future total demand of the product because it is also path dependent. This path dependency is caused by the time value of money and the steep learning curve in building jumbo-jets, where the first few planes can cost five to six times the cost of the one-hundredth plane (Benkard 2000).

As a practical matter, we can solve the game-theory part of the problem through the joint use of simulations and a common technique used to solve game theoretic problems, "backward induction." For example, under certain assumed conditions, a static picture of the tradeoff for one decision period is illustrated in Table 1.

Table 1 shows net present value (NPV) profit results of the launch or no launch options for Boeing and Airbus at their Nash Equilibrium points. Looking forward, the game matrix shows Airbus is always better off launching regardless of what Boeing decides to do. This means that Boeing needs to

		Boeing	
		No Launch	*Launch*
Airbus	*No Launch*	7,421 0	12,302 0
	Launch	2,729 4,132	2,391 3,896

Figure 1-1 A 2x2 matrix of NPV payoffs

base its their decision on the assumption that Airbus will act rationally and launch. Consequently, under these conditions Boeing will decide not to launch, despite the fact that they could make over $12 Billion if they launch and Airbus does not, since doing so maximizes their profit when Airbus launches.

Traditionally, there are several ways of valuing a real option, such as partial differential equations, dynamic programming, or Monte Carlo simulations (Dixit and Pindyck 1994; Trigeorgis 1995). In the Monte Carlo technique, one first generates a random series of observations according to the estimated distributions of the variables thought to affect the payoffs of the given investment, and then calculates the cashflows for each period. One then calculates the net present value of that cash flow stream. By generating a large sample of such simulated cashflow streams and taking the average of their net present value, one can arrive at the value of the real option.

Game theoretic reasoning can be incorporated into this analysis by deriving the optimal strategy for each firm over the entire sequence by utilizing "backward induction" (Ghemawat 1991). For every random sequence generated, the optimal strategy for the firm is derived by iteratively determining the optimal strategy at each stage of the "game" beginning with the final period and working backward. This procedure ensures that each player takes the optimal action for that particular realization of the random process. By generating a large sample of such random paths with optimal actions over each path and calculating the average net present value over the whole sample, one can determine the optimal strategy / investment decision for the firm. This approach to analyzing such a decision incorporates both the "commitment" value of the investment as well as its options value.

The assumption that demand is stochastic enables this model to be created. Demand is assumed to follow a Wiener process with a normal distribution around the last periods demand realizations occurring at yearly intervals. This creates a Markov Chain similar to what has been proven to be a good representation the airline industry (Benkard 2000).

The stochastic demand assumption is now inserted into a program that dynamically sets market share, plane prices, etc depending upon the conditions of demand and the entry of the two airlines. The program then generates an NPV for each scenario. The results are captured in two 21x21 matrices, one each for Boeing and Airbus, with the rows and columns representing the years that Airbus and Boeing enter respectively as shown in appendix 1. For example cell (3, 5) of the matrix would correspond to the scenario where Airbus enters in year 3 and Boeing in year 5 for the demand generated for that realization.

1.3 Problem Statement

Despite the rapid incorporation of game theory and real options into the academic fields of strategy, operations and corporate finance, little progress has occurred in the transfer of the resulting analytical tools into practice. Unfortunately none of this research helps a manager that is drowning in a sea of uncertainty. The few methods of how game-theoretic and options-theoretic reasoning could be usefully integrated together in the analysis of strategic decisions that have been developed have taken an approach that is too academic and theoretical to have any use to a manger under pressure to make a decision.

This study evaluates and develops methods to jointly incorporate game theory and real options analysis into a decision making tool that a manger can easily, and quickly use to make real time decisions.

This study evaluates the feasibility of using a Hazard model to predict the optimal time until launch in a similar way to how a Hazard model is used in fields such as medicine and insurance that use a Hazard model to predict time until an event such as sickness or death. The limitations and proper use of Hazard models will be set, and the validity of using a Hazard model evaluated.

Then a methodology of using either a hazard model or what ever the study finds to be the best way to evaluate the interface of real options and game theory in new product introduction decisions will be introduced, validated and the details of how a manager needing to make a decision can to use this method to make better decisions given.

In conclusion, this study makes three key contributions.

1. First, it outlines an approach to integrating game-theoretic and options-theoretic reasoning to the strategic analysis of new product development that can be used to make real time entry decisions. Over the past twenty years, these two approaches have exercised increasing influence on the field's understanding of strategic choice, but the useful integration of the two approaches has not occurred.
2. Second, the possible use of a Hazard model for real time predictions is evaluated. Hazard regression is commonly used and accepted as the way to

regress real options. This study will explore whether or not a hazard regression can then be used to model the probability of the event of optimal entry for real time decision making.

3. Third, this study develops a method that can be practically implemented by managers under pressure to make a good decision.

1.4 Delimitations

The model is of a two player game and cannot handle the complexity of multiple player games. This is typical of game theoretical models. However, in many cases it is a simple matter to lump the competition together and model them as a single entity without changing the results of the model beyond reason. Thus, the model will work for general NPI's that have similar industry structure and not just VLA's. Further, the key contribution of a game-theory/real-option methodology can be applied to not only NPI but also other decisions that face a real options and game theoretic decision, which happens to be almost all major decisions

CHAPTER 2. LITERATURE REVIEW

Literature on new product development is diverse with areas focusing on both the how and the why. The purpose of this chapter is to provide the necessary background to enable the reader to understand the importance and direction of new product development. Major prevalent themes are presented with emphasis of tying the why and how of product development from a strategic point of view instead of the numerous possible tactics that can be used.

2.1 Importance of New Product Development

We are in an age where speeding products to market has become paramount to a firms success. Product lifecycles are now often measured in months instead of years putting incredible pressure on shortening the product development cycle time. For many firms the ability to gain and sustain a competitive advantage lies in faster product development cycle time as new products are increasingly becoming the nexus of competition in many technology- and R&D-intensive industries (Clark and Fujimoto 1991; Brown and Eisenhardt 1995). Product development is also a strategic means for a company to diversify, and/or alter focus to adapt to changing market conditions (Schoonhoven, Eisenhardt, & Lyman, 1990).

2.2 Why Develop Products Faster

This section will look at advantages that can be gained and the strategic motivation of shortening the development time of new product development,

2.2.1 Quick product development time

As a strategic weapon time is an equivalent with money, quality productivity and innovation as a source of competitive advantage (Stalk 1988). Preston G. Smith and Donald G. Reinertsein [1991] discuss why a company would want to develop products faster. They argue that while different companies' motivations will vary the following are general principles that drive for fast development time:

1. Increased Sales – Each month that can be cut from development is month that can be added to its sales. The sales life of the product is not only extended backwards but forwards in instances where loyalty due to switching costs creating early momentum allowing the product to remain on the market longer.
2. Higher Margins – in many products the price a customer is willing to pay is decreasing as a function of time. Also, the sooner a product is released the probability of more pricing freedom increases as there is less competition. These factors allow new products to have higher margins during their early stages compared to latter more mature market.
3. Surprising the competition – in the dynamic world of new products early introduction can surprise the competition and change the market conditions.

4. Responsiveness to Changing Markets, Styles and Technologies – with the fast pace of changing technology a strong old line of products can be made obsolete quite abruptly. If a company can not respond quickly revenue and reputation can be lost. Styling is also important. Chrysler has recently enjoyed success because its United States competitors' vehicles often look dated by the time that they are introduced to the market. A fast-cycle time leads to flexibility to take advantage of or minimize the downside of change.
5. Maintain a Market Leadership Position – Many companies are known for being on the cutting edge of technology and the forefront of their marketplace. Companies such as Honda, Hewlett-Packard and Sony are seen as trend setters and customers are willing to follow trends set by these companies and pay more for their new products. Many companies regard accelerated development as their core competency.

2.2.2 First mover advantage

One of the forces behind fast product development is the strategic advantage of being first to market (Stalk, 1988) Figure 2-1 shows a framework that Lieberman and Montgomery (1988) presented as illustrating how first mover advantages lead to profits.

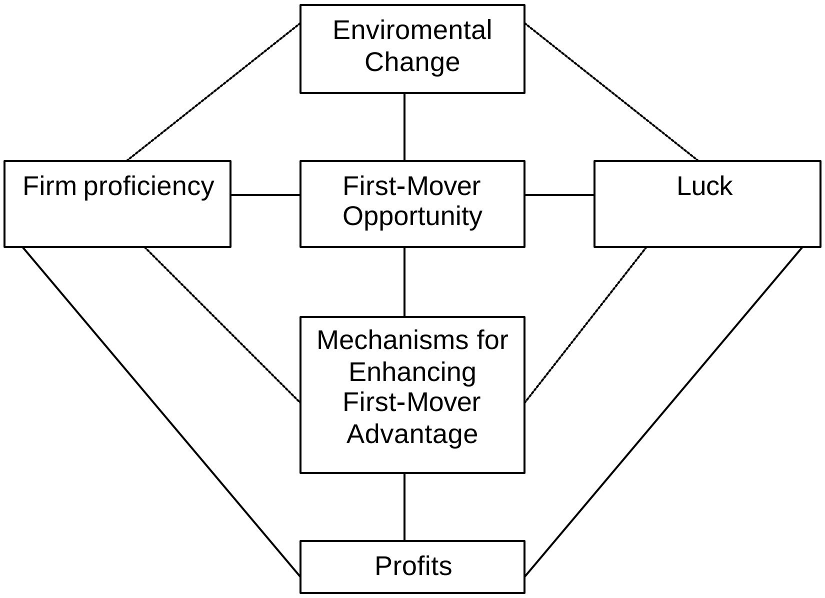


Figure 2-1: Endogenous generation of first-mover advantages.

Lieberman and Montgomery (1988, 1998) present that first mover advantages come from three primary sources. They would argue that previously mentioned advantages stem from the following:

1. Technological Leadership – there are two main mechanisms by which advantages can be gained in technological leadership.
    a. Advantages derived from the learning or experience curve where prices fall with cumulative output. In the 1970 the Boston Consulting Group popularized the idea of gaining advantages through the learning curve. By being first to market a company can position itself further down the learning curve than competitors giving a competitive advantage in many industries.

    b. Success in Patent or R&D races. In many industries such as pharmaceuticals the winner of patent or trade secret R&D races is the first to market securing market position. More recently first movers have been shown to have an advantage with respect to influencing the path of dominant design, which is often path dependent due to switching costs and other factors. (Suarez and Utterback 1995)

2. Preemption of Assets(resources) – the first mover can gain advantage by obtaining control of existing assets. These assets can be broken down into the following three areas.
    a. Input factors such as natural resource deposits can often be gained at market prices below the future market evolution inflates them.
    b. Location in geographic and product characteristics can be a sustained advantage if there is limited "room" whether physically or economically. Often the "bottleneck" of an industry can be controlled in this way similar to how Coke and Pepsi dominate distribution channels in the soft drink industry.
    c. Plant and equipment advantages can be sustainable when scale economies can deter entrants
3. Buyer Switching Costs – both switching costs and buyer uncertainty can give first mover advantages where late entrants must invest extra resources to attract customers away from the original.

2.3 Costs of Speed

It can be very expensive and inefficient to develop products too quickly (Smith and Reinerstsen 1998). Time is not free. To introduce a product sooner a company has to be willing to make the tradeoffs for time. These tradeoffs come in many forms such as inferior product design, increased expenses due to time compression diseconomies of scale etc.

2.3.1 Time cost tradeoff

Observations have shown that there exists a U-shaped relationship between time and the total development cost. The typical company is on the right side of the minimum of this curve. They can easily reduce their costs and time by moving further down the curve (Gupta, Brockhoff, and Weisenfeld, 1992; Smith and Reinertsen, 1998; Bayus, 1997). While the typical company has this opportunity, most believe that they are operating on the left side of the minimum (Gupta, Brockhoff, and Weisenfeld, 1992). Figure 2-2 graphically shows this tradeoff.

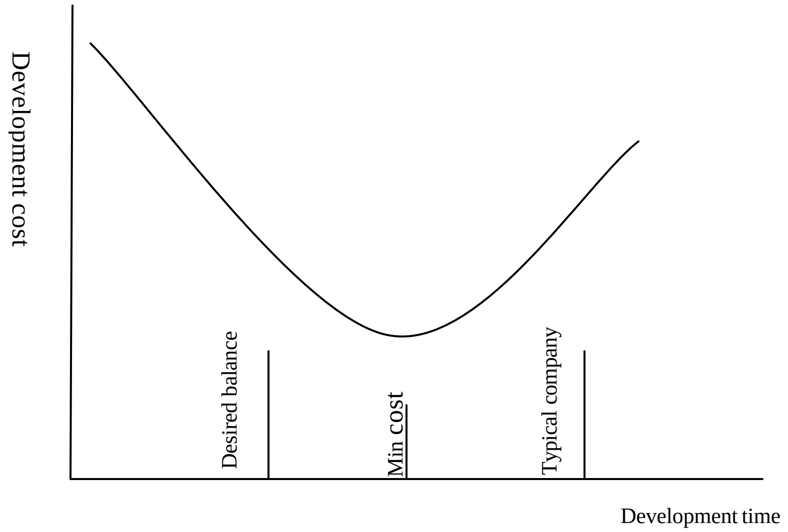


Figure 2-2: The development time cost tradeoff. Adapted from Bayus (1997)

Bayus (1997) modeled this time cost tradeoff curve and showed that optimal time to market is really a function of the product and market conditions. Bayus (1997) then developed a speed-to-market model optimizing new product decisions and the associated markets, demand, and cost conditions.

2.3.2 Competing objectives.

In the product development process there are multiple objectives that compete with each other. In order to further one objective another needs to be sacrificed. Managers have intuitively known and stated this in the common phrase: “Good, fast cheap … Pick any two” (Bayus 1997). However, the problem is actually more complicated than this. Figure 2-3 shows four key product development objectives and the six corresponding tradeoffs (Smith and Reinerstsen 1998).

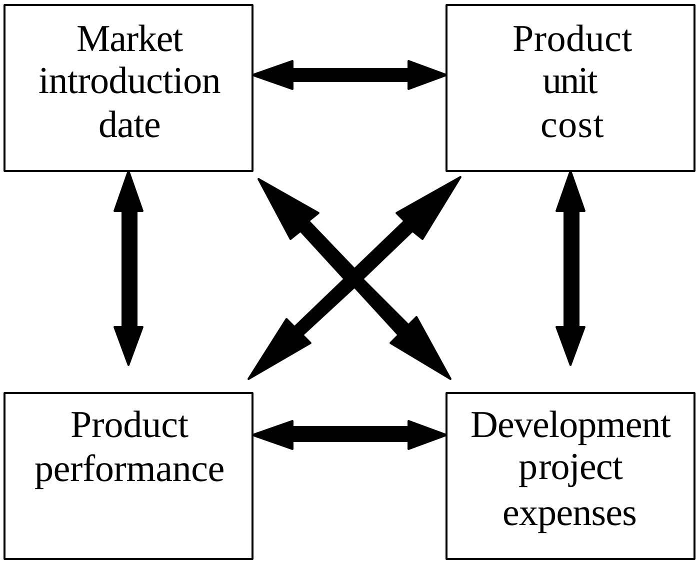


Figure 2-3: Four key product development objectives and six tradeoffs.

In order to balance these objectives managers need to remember that the overriding objective is not any one of these, nor a specific combination of them, but to make money. Cost in these models is not just strictly monetary but the opportunity costs of enhanced product performance, loss of flexibility etc that have an impact on the bottom line. Optimizing on only one of these tradeoffs will lead to failure. In order to make good decision, decision rules are needed based on financial markets (Smith and Reinertsen 1998).

Because of the importance of making good decisions in the face of conflicting objectives, models have been developed to measure these tradeoffs. The previously mentioned Bayus model modeled two competitive scenarios. In the first scenario a firm needs to decide whether to accelerate development to catch a competitor that has recently introduced a new product. The second scenario is where a firm needs to decide whether or not to speed development to beat the competition to market.

Cohen, Eliashberg and Ho (1996) developed a product performance and time-to-market trade off model that showed minimizing breakeven time can lead to premature product introduction. The model uses a multistage product performance improvement process of, Design → Process → Market, to study how different resources should be allocated over the different stages. It also considered the cumulative costs and revenues of the new product over its entire life cycle. The model mainly focuses on the marketing aspect of product development and improving product characteristics and performance. The model shows that often it is better to take time to develop a superior product and improved product development capability should not and is not always demonstrated by earlier time to market but always leads to enhanced products.

Some research in new product development has changed its focus from having an emphasis of speed to market towards the market tradeoff for optimal performance. These models are representative of this change in focus.

While most of these models examine the external forces that dictate optimal product development time frames by measuring opportunity costs as product costs they really are about product positioning. Very little research has been about the costs associated with the design of the process involved in making the product.

2.4 *Time-to-Market Tradeoff*

Although time-to-market has become a major focus of many large companies, being first to market and the fastest in development is not always better (Lambert and Slater 1999).

2.4.1 First-mover disadvantages

Early commercialization does not guarantee a position of strength in the market. The experiences of EMI in CT scanners and Xerox in personal computers illustrate the challenges faced by many first movers that failed to earn competitive advantage or even survive (Teece 1986). Lieberman and Montgomery (1988,1998) point out some first mover disadvantages. Late movers can enjoy advantages such as: (1) free-riding on the first mover's investments, (2) technological and market uncertainty, (3) technological discontinuities, (4) incumbent inertia of the first mover making it difficult to adapt to change.

2.4.2 Market timing

A market's readiness to receive a new product is also not constant. Abell (1978) introduced the concept of a "strategic window of opportunity". A new product's success depends on its timing. Entry which is too late represents lost opportunity; on the other hand, a product introduced to the market too early may not be received by customers or market channels. There are many examples of products that were great success stories in the 90's that were first unsuccessfully introduced in the 80's. A company needs to be aware of market conditions and in a position to take advantage of opportunities that present themselves.

Upon realizing that being first is not everything, Dacko, Furrer, Liu, Sudharshan (2001) showed that many markets have rhythms and suggested an approach of matching product introduction and development to the rhythm of the market. This research shows that the internal development timing question is partly a function of the market.

2.5 New Product Evaluation

The decision of whether or not funding should be allocated for a new product is almost always justified through a discounted cash flow analysis (DCF). Not only is DCF a inferior method to evaluate the true value of an investment, the optimal launch date is dependent upon more than a positive cash flow as previously discussed. Real options can be used to evaluate the timing of launching a new product under market uncertainty.

2.5.1 Discounted cash flow

Probably the most common project evaluation method is the net present value (NPV) method. However, NPV and other DCF evaluation methods are recognized to be inadequate approaches to capital budgeting. This is because they cannot properly capture the value of flexibility to adapt and revise later decisions in response to unknown market developments. Unfortunately the only constant in the business world is uncertainty, making NPV calculations inevitably wrong since NPV calculations make implicit assumptions creating an "expected scenario" with its respective cash flows.

(Trigeorgis 1995)

Despite its imperfections tradition NPV should not be abandoned. Trigeorgis (1995) suggests that traditional NPV methods should be expanded to include the option value of the investment, i.e., Expanded (strategic) NPV = static (passive} NPV of expected cash flows

+ value of options from active management.

The methods of how to evaluate the value of the option have been thoroughly debated in recent literature. By using the methods that have been established a more correct evaluation can be made concerning the value of a project.

2.5.2 Real options

The quantitative underpinnings of options derive from the pricing of financial options. The Black and Scholes equation (Black and Scholes 1973) formally introduced a risk free way to price financial options. This equation was derived using stochastic calculus and partial differential equations. Since then other methods have been explored because defining a set of partial differential equations may not even be possible, let alone find a closed form solution when dealing with more typical real life applications such as when there are multiple options interacting (Trigeorgis 1995).

Various numerical analysis techniques have been developed to evaluate options under complicated conditions. Trigeorgis breaks these methods into two different numerical techniques:

1. Those that approximate the underlying stochastic processes directly and are generally more intuitive
2. Those approximating the resulting partial differential equations.

Monte Carlo simulation (Boyle 1977), various lattice approaches such as Cox, Ross, and Rubinstein's (1979) standard binomial lattice method, and Trigeorgis' logtransformed binomial method all fit into the first category. The second category includes numerical integration, and implicit or explicit finite difference schemes.

Trigeorgis also lists categories of the common applications of real options. They are:

1. Option to defer—Management has an option to invest, so it can wait x years to see if conditions justify the investment. An example is an option to buy land in real-estate development.
2. Time-to-build or staged investment option—Each stage in an investment can be viewed as an option on the value of subsequent stages.
3. Option to alter operating scale—Under changing market conditions a firm can expand, contract, shut down and/or restart.
4. Option to abandon—Permanent termination of operations realizing the resale value of assets.
5. Option to switch—Outputs can be changed giving product flexibility, or the same outputs can be produced with different inputs giving process flexibility.
6. Growth options—Where an earlier investment is a prerequisite or a link in a chain of unrelated products or markets that open up future growth opportunities.
7. Multiple interacting options—Most real life projects include a collection of the options listed above.

Common to all real options is the value of deferring a decision. Merton (1998) points out that:

> The common element for using option-pricing here is . . . [that] the future is uncertain (if it were not, there would be no need to create options because we know now what we will do later) and in an uncertain environment, having the flexibility to decide what to do after some of that uncertainty is resolved definitely has value (1998: 339).

Even though forecasting techniques are improving, uncertainty is most likely increasing along with the rapid pace of technology. Thus, the value of using real options in project evaluation is more valuable than ever before. If a firm is going to be successful in maximizing their profit of new products it is necessary that they use a real options approach to capture the value of flexibility under uncertainty.

2.5.3 Options and new product development

New product development already currently utilizes methods that capture the value of these options.

Pharmaceuticals and other R&D intense industries heavily leverage the time-to-build option. In fact pharmaceutical companies have failure rates of 90-95 percent of projects with most ending in the early or middle stages of development (Ittner and Kogut 1995).

Another common use of options thinking is when companies try to mitigate the risk and problems of new process development is the use of modules. Modularity can help firms compete by promoting time-pacing (Brown and Eisenhardt 1998), managing complexity (Baldwin and Clark 1997), enabling economies of substitution (Garud and Kumaraswamy 1995), increasing firms’ strategic flexibility to respond to environmental change (Sanchez and Mahoney 1996) and/or more effectively manage the tradeoff of switching from process development to manufacturing, improving performance (Hatch and Macher 2002). A module is effectively an option on future development and flexibility. Car and computer companies build “platforms” at an increased cost that allow for modularity, which can be well understood as real options (Baldwin & Clark, 2000).

Baldwin and Clark (2000) have also assessed the tradeoff of whether the investment to create modularity in production is worth the additional complexity of the design which is really just a question of whether the value of the options is greater than the increased cost of complexity. Mcgrath (1997) has also shown real options are toehold investments designed to better prepare the investor to meet uncertain events in the future (McGrath, 1997).

2.6 Competitive Response and Game Theory

One of the biggest contributors to market uncertainty is competitor response. By combining real options with a game-theoretical approach the timing decision can more fully evaluate when the optimal launch date is, and determine what factors influence when this date occurs.

2.6.1 Game theory

The first studies of games were done on Oligopoly pricing and production. Cournot (1838), Bertrand (1883) and Edgeworth (1897) all explored how firms in an oligopoly would choose pricing and production levels. However, these were seen as special cases and the not applicable in other circumstances. Von Neumann (1928) then built upon this work in 1928 when he proved the minimax theorem which has been a central concept of game theory. Neumann (1944) then collaborated with Morgenstern to publish Theory of Games and Economic Behavior, which was the first time game theory had been brought into the spotlight.

In 1950 Nash introduced the idea of a non-cooperative solution where each player maximizes their payoff given the other players' strategies extending game theory to non-zero-sum games. A non-zero-sum game acknowledges the possibility that in a 2 player game both players could win or both could lose. The resulting solution of the players' strategies is called the Nash equilibrium.

The classic example of this is the prisoners' dilemma. In this situation there are two prisoners that are being questioned separately. If they both lie, they get away free. However, the warden offers a lighter punishment to each if they rat and the other does not. Unfortunately, the Nash equilibrium leads both to rat, and they both end up worse off for it.

Using the foundational work discussed game theory has come to dominate much of modern economics and been widely used in many fields. For example it is used in biology to predict animal behavior and in law to settle bankruptcy settlements (Fudenberg and Tirole 1985). In fact, game theory has been widely applied to evolutionary concepts both in biology and the social sciences to the extent that in the preface to Evolution and the Theory of Games, Maynard Smith (1982) states, "it has turned out that game theory is more readily applied to biology than to the field of economic behaviour for which it was originally designed."

In Courtney's (2000) Games managers should play he states that there are five elements of competitive intelligence that need to be understood in order to create a game that is an accurate representation of any situation. These five elements of the game are:

1. Define the Strategic Issue –What decision are you trying to make and how is it related to other other internal and external decision
2. Determine the relevant players—Which players will have impact upon the success of your strategy
3. Identify each player's strategic objectives—it may or not be profit maximizing, for example the player may only be after market share, or short run returns etc
4. Identify the potential actions for each player—with each player's strategic motives in mind determine what possible action they might take under the different circumstances created by the game.
5. Determine the likely structure of the game—Will decisions be made sequentially, simultaneously, is the game repeated etc

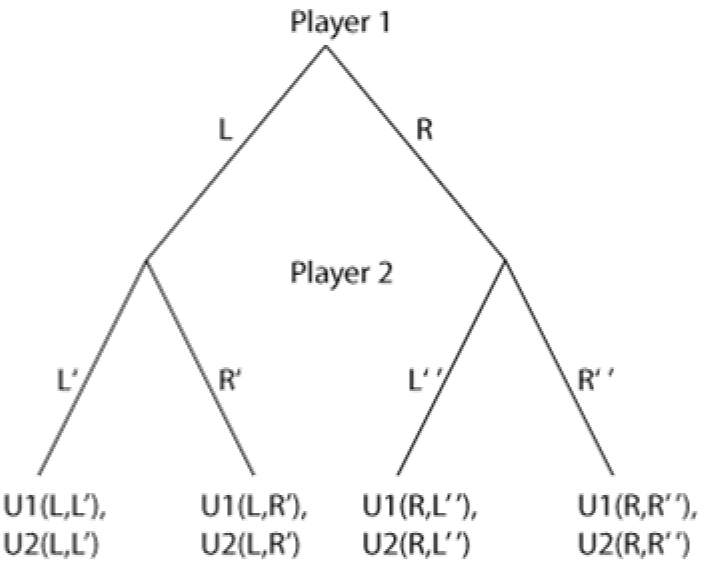
After these five elements are determined market research can provide the payouts for each scenario and the game evaluated.

Once the game has been defined it can be represented in any of following three forms:

1. Extensive or tree form
2. Matrix form
3. Characteristic function form

Each of these forms provides different levels of detail. The extensive form is the most detailed and consists of a complete formal description of the game played including sequencing of moves, necessary knowledge at each node, any random occurrences, and the payoffs to each player. The matrix form contains less information, and the characteristic form the least of all providing only information about the payoffs. A graphical representation of each of these three forms is provided in Figure 2.4.

Extensive/Tree form



The game begins with a decision node for player 1, where 1 chooses between L and R. player 1 chooses L, then a decision node for player 2 is reached, where 2 chooses between L' and R'. Likewise, if player 1 chooses R then another decision node for player 2 is reached, where 2 chooses between L" and R". Following each of player 2's choices, a terminal node is reached (i.e., the game ends) and the indicated payoffs are received. (http://www.econport.org:8080/econport/request?page=man_gametheory_dyngames)

Strategic/Matrix form

		Boeing	
		No Launch	*Launch*
Airbus	*No Launch*	7,421 0	12,302 0
	Launch	2,729 4,132	2,391 3,896

A 2x2 matrix showing the NPV payoffs in Millions of dollars for Boeing and Airbus

Characteristic function form

$$v_i(s) = max \left\{ S\ l^i x^i,\ x \in V(S) \right\}$$

where *x* represents an outcome,

v the value of the game &

λ an arbitrary weight

Figure 2.4 Three representative forms of a game

2.6.2 Timing of technology introduction in a duolpoly

Scherer (1967) evaluated the introduction of a product in a duopoly. In his study the firms were identical and found that if they were required to pre-commit themselves that they would both enter as soon as possible which was earlier than the optimal time.

Reinganum (1981 a,b) showed that there must be a technology diffusion of technology forcing the firms to effectively enter on different dates even though they are identical and there is no uncertainty.

Fudenberg and Tirole (1985) later showed that identical firms that follow a diffusion process will always be forced to face preemption, and thus force both to equal payoffs in equilibrium.

2.6.3 Integrating real options and game theory

Smit and Ankum (1993) offered a game-theoretic treatment of competitive reactions under various market structures using a real options framework. They actual embed a two player game into each node of a decision tree. However, the more complicated N-person game has not yet been solved.

Grenadier (1996) developed a equilibrium framework for strategic option exercise games, focusing on the real estate market. In 2000 Grenadier edited Game Choices: The Intersection of Real Option and Game Theory, in which he compiles what little work that has been done in this area. There he states that real options research generally assumes that the exercising of an option has no effect on the value of other agents' options and that assumption is not consistent with reality. Unfortunately, the intersection of these two methodologies is still in their infancy. On the other hand, as Grenadier (2000) states, "It will be exciting to see the future trajectory of research in this area in the coming years."

Even since 2000 when grenadier made that statement the rapid incorporation of game theory and real options has been limited in the transfer of the resulting analytical tools into practice. Moreover, little attention has been paid to how game-theoretic and options-theoretic reasoning could be usefully integrated together in the analysis of strategic decisions (Adner and Levinthal 2004).

CHAPTER 3. METHODOLGY FOR MODELING THE INVESTMENT DECISION

The traditional method for evaluating the attractiveness of investing in a new technology or market opportunity for a new product is discounted cash flow analysis. However, when there are rivals contemplating the same decision, the decisions of one firm will influence the performance of its rivals in addition to its own performance. Ignoring the decisions of rivals would likely lead to incorrect estimates of the firm's market share, revenues, and discounted cash flows. To explicitly account for the interdependence between the firms' decisions, this study constructs a game theoretic model of the decision to invest in the development and launch a new product. In addition to the complications of interdependent cash flows, it is common for such a decision to be fraught with uncertainty. In this case, demand for the new product is assumed to be unknown and volatile. In the face of this great uncertainty into the model, the model integrates real options analysis with the game theoretic analysis to make the launch decision. More specifically, the model measures the value of the option to defer investment and learn more about the underlying level of demand.

3.1 Game-Theoretic Analysis

If the uncertain demand can possibly fall to levels that render the investment unprofitable, the firm may prefer to delay the launch until a profitable level of demand can be verified. To accommodate this possibility of deferring investment, the model is constructed of a two-player (Airbus and Boeing), multi-staged, sequential game for the game-theoretic element of the analysis. Each firm is restricted to invest and launch its new product within a fixed time frame (*n* years) and it is assumed that if either firm has not entered within that time it has committed to not enter. Therefore, each firm is independently able to choose to enter in any one of the years in the n-year time frame. These decisions can be represented in a normal-form game that is constructed in an (*n* + 1) x (*n* + 1) matrix (period *n* + 1 indicates commitment to not launch). This entry game can be seen as a (2 · *n*) stage extensive-form game of launch/no launch decisions where each year is represented in two stages (that year's choices by Airbus and Boeing) of simultaneous moves. Since firms are not allowed to exit in the game, many branches of the extensive-form disappear when entry occurs in early stages. For example, if both firms enter in the first period (stages one and two), the decision in the second period of whether to launch or not launch is moot. The use of the normal-form game of dimension (*n* + 1) x (*n* + 1) is a collapsing of the complete set of branches down to the feasible set of branches.

The payoffs for each cell of the (*n*+1) x (*n*+1) normal-form game come from discounted cash flow analysis of that particular launch scenario. To see this, consider the stylized normal-form game in Table 3.1. The payoffs in cell (2, 4) are $?_{A|\,2,4}$ and $?_{B|\,2,4}$ and result from Airbus' decision to enter in the second period and Boeing's decision to enter in the fourth period. The payoffs are the discounted cash flows derived from the competition between Airbus and Boeing defined by the specific scenario in each given year. For example, in cell (2,4) the cash flows $(?_{A¦\,2,4}, ?_{B¦\,2,4})$ are constructed assuming Boeing is a monopolist with the 747 until year two when Airbus invests, then Airbus competes as a duopolist against the 747 until Boeing invests in the fourth period afterwhich Airbus competes as a duopolist against the 747X.

When both firms are present in the market, the model assumes that they are competing in a differentiated Bertrand oligopoly where each firm's revenue is influenced by the pricing decision of its rival. This implies that every cell of the normal-form game embeds an underlying, sequential pricing game defined by the oligopolistic competitive environment in each year.

Table 3-1. Normal form entry game with a 20 year horizon

Airbus Entry Dates	Boeing Entry Dates				
	1	2	…	20	21
1	$(?_{A¦1,1}, ?_{B¦1,1})$	$(?_{A¦1,2}, ?_{B¦1,2})$	…	$(?_{A¦\,1,20}, ?_{B¦\,1,20})$	$(?_{A¦1,21}, ?_{B¦1,21})$
2	$(?_{A¦2,1}, ?_{B¦2,1})$	$(?_{A¦2,2}, ?_{B¦2,2})$	…	$(?_{A¦\,2,20}, ?_{B¦\,2,20})$	$(?_{A¦2,21}, ?_{B¦2,21})$
3	$(?_{A¦3,1}, ?_{B¦3,1})$	$(?_{A¦3,2}, ?_{B¦3,2})$	…	$(?_{A¦\,3,20}, ?_{B¦\,3,20})$	$(?_{A¦3,21}, ?_{B¦3,21})$
⋮	⋮	⋮	⋱	⋮	⋮
20	$(?_{A¦20,1}, ?_{B¦\,20,1})$	$(?_{A¦20,2}, ?_{B¦20,2})$	…	$(?_{A¦\,1,20}, ?_{B¦\,1,20})$	$(?_{A¦1,21}, ?_{B¦1,21})$
21	$(?_{A¦20,1}, ?_{B¦\,21,1})$	$(?_{A¦21,2}, ?_{B¦21,2})$	…	$(?_{A¦21,20}, ?_{B¦\,21,20})$	$(?_{A¦21,21}, ?_{B¦21,21})$

To model the underlying differentiated Bertrand pricing games, the study begins by specifying the revenue and cost functions for each firm's annual objective function. The revenue function in period t for Airbus is

$$R_{At} = R(P_{At}; P_{Bt}) \tag{1}$$

where P_{At} is the price of the A380 in period *t* and P_{Bit} is the price of the Boeing 747 or 747X (distinguished by the subscript *i*) in period t. For the sake of solving the Bertrand pricing game Airbus' cost function is specified in period t as a function of Airbus' quantity which is a function of Airbus' price:

$$C_{At} = C(Q_{At}(P_{At})): \tag{2}$$

Based on the revenue and cost functions, Airbus' problem is to choose the profit maximizing price in period t:

$$\max_{P_{At}} \mathrm{p}_{At} = R(P_{At}; P_{Bt}) - C(Q_{At}(P_{At}; P_{Bt})) \tag{3}$$

Deriving the first order conditions of Airbus' problem and solving for Airbus' profit maximizing price gives us

$$P_{At} = r_{At}(P_{Bt}) \tag{4}$$

where $r_{At}(P_{Bt})$ is the classic Bertrand reaction function. Because Boeing's price is embedded in Airbus' revenue function, Airbus' optimal price is an increasing function of Boeing's price. The reaction function $r_{At}(P_{Bt})$ gives an infinite set of prices that are Airbus' best response to all possible prices set by Boeing. The remaining question for Airbus is where Boeing will set its price. Boeing's profit maximization problem is similar to that of Airbus:

$$\max_{P_{Bt}} p_{Bt} = R(P_{At}; P_{Bt}) - C(Q_{Bt}(P_{Bt})) \tag{5}$$

Solving for the first order conditions will give Boeing's optimal price as a function of Airbus' choice of price:

$$P_{Bt} = r_{Bt}(P_{At}) \tag{6}$$

and using the realized demand for that period.

Since both firms insist on producing on their reaction functions, the only place that an equilibrium can exist is where the reaction functions cross. This crossing point is found by solving the system of two equations in two unknowns $(r_{At}(P_{Bt}), r_{Bt}(P_{At}))$ and finally obtain the Nash equilibrium pair of prices that are the solution to the pricing game in period t, (P^*_{At}, P^*_{Bt}). Substituting these equilibrium prices into each firm's profit function gives the optimal profit for each firm in period t. Each optimal profit is a single entry into that firm's discounted cash flow for a particular entry decision.

Of course, to complete the payoff for a cell of the entry game, the equilibrium prices and resulting profits for every period in the time horizon are needed. Since it is necessary to populate every cell in the $(n + 1)$ x $(n + 1)$ normal-form game, there will be very few cases where the competitive environment remains constant throughout the time horizon. The payoffs for a single cell could comprise periods of monopoly (Boeing 747), duopoly with A380 and 747, and duopoly with A380 and 747X. Therefore, the payoffs are the discounted sum of a stream of annual profits based on annual equilibrium prices and the specific entry conditions of each period. The timing of investment for each firm defines the particular competitive environment for each year. Every cell in the normal-form game comprises a sequence of competitive environments defined by the particular investment timing implied in that cell. Let t_A be the timing of Airbus' investment and t_B be the timing of Boeing's investment. Then, the payoffs in each cell are the discounted sum of a stream of annual profits based on annual equilibrium prices and the specific entry conditions of each period:

$$? \,_{A|}\, t_A, t_B = \sum_{t=1}^{n} \frac{1}{(1+r)^t} [R(P^*_{At}, P^*_{Bt} | \; t_A, t_B) - C\,(Q_{At}(P_{At}))] \qquad (7)$$

$$? \,_{B|}\, t_A, t_B = \sum_{t=1}^{n} \frac{1}{(1+r)^t} [R(P^*_{At}, P^*_{Bt} | \; t_A, t_B) - C\,(Q_{Bt}(P_{Bt}))] \qquad (8)$$

The firms maximize their discounted stream of profits by choosing a series of Nash equilibrium prices given each particular entry date. This results in optimal discounted cash flows ($? \,_{A|}\, t_A, t_B$, $? \,_{A|}\, t_A, t_B$) that are the payoffs for cell (t_A, t_B) in the normal-form entry game.

Having specified the conditional payoffs for each player under all possible actions, the model determines each firm's strategy. Airbus' strategy is its complete set of optimal timing decisions in response to Boeing investing in every possible period. For all t_B from period one to (n+1), Airbus' decision is

$$\max_{t_A} \; ? \,_{A|}\, _{tB=1}\,(tA) = \sum_{t=1}^{n} \frac{1}{(1+r)^t} [R(P^*_{At}, P^*_{Bt}, tA | \; tB = 1) - C\,(Q_{At}(P_{At}))]$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$\max_{t_A} \; ? \,_{A|}\, _{tB=n+1}\,(tA) = \sum_{t=1}^{n} \frac{1}{(1+r)^t} [R(P^*_{At}, P^*_{Bt}, tA | \; tB = n+1) - C\,(Q_{At}(P_{At}))]$$

The model finds Boeing's strategy in like manner. Given the strategy of each firm, the model finds the Nash equilibrium for the investment decision by determining which investment dates are simultaneous best responses for Airbus and Boeing. Of course, in practice there may be no equilibrium or multiple equilibria.

3.2 Real Options Analysis

Traditional approaches to valuing the Airbus A380 project would attempt to evaluate the discounted cash flows of the project. This study employs the game theoretic model to overcome the challenge that Airbus’ cash flows will depend on Boeing’s entry and pricing decisions and Boeing will be similarly influenced by Airbus. However, game theory alone is not enough to fully model the decision each firm faces because each firm holds a real option to delay entry to resolve some of the ex ante uncertainty regarding the size of the market. An integrated model of game theory and real options is required to make the decision of whether and when to enter. With this integration, the model can evaluate the entry decision while facing great uncertainty and a competitive rival.

The essence of the value of a real option when facing uncertainty is the opportunity for the firm to resolve some of the uncertainty before making its irreversible investment (Copeland and Antikarnov 2001, Dixit and Pindyck 1994, Trigeorgis 1996). If the firm learns that the uncertain variable will lead to cash flows below a critical value, the firm will simply not invest (Adner and Levinthal 2004). Thus, in exchange for the upfront expense of the option, the firm is able to reduce or even eliminate the downside risk while still preserving the upside risk of the project.

In contrast, traditional net present value analysis assumes that the investment will happen immediately and makes no allowance for learning of an unprofitable realization of the uncertain variable. Net present value analysis takes the a priori expectation of uncertain cash flows, including cash flows that lead to negative profits that the firm would avoid if it could. Therefore, net present value analysis explicitly incorporates the possibility of unprofitable outcomes while real options analysis explicitly eliminates or at least reduces the probability of the same unprofitable outcomes. Of course, the value of the real option relies on the ability to resolve at least some of the uncertainty. If uncertainty can not be resolved, the real option has no value.

Consider the problem of uncertain demand for superjumbo aircraft. In the unlikely case that the uncertain demand is known ex ante to be within a range that is high enough to ensure that both firms can profitably enter, all firms will invest immediately to capture the early cash flow that would have been lost if the investment were deferred (Smit and Ankum 1993). In the more likely case that the distribution of the uncertain demand allows the ex post realizations of demand to fall to levels that earn negative net present value for at least one firm, the investment decision must include analysis of whether to defer investment to better learn the realized level of demand. When the true demand is found to be below the critical value, the project is abandoned and the firm loses only the cost of acquiring and holding the option to defer. When the true demand is found to be above the critical value, the firm “sells” the option to defer and invests with certainty, or at least higher probability, in a profitable outcome. Early on in the specific realization of demand, the low level and downward trend of demand bodes ill for the project. However, through the option to delay, demand can be observed to be sufficient to profitably invest.

Combining game-theoretic and real-options approaches is problematic because of differences in the underlying logic of the two perspectives. For example, game-theory and real options differ in how they characterize the interrelationships between individual action and the external industry environment. In game theory, current industry conditions are largely characterized as resulting from the past actions taken by industry players; while in real options, industry conditions are modeled as the outcome of random stochastic processes. In other words, industry conditions are endogenous in game theoretic models and exogenous in real options. Integrating game theory and real options is made more difficult because payoffs can vary depending upon the actions taken by the players as well as the realizations of stochastic processes. This aspect of the problem is not normally featured in real options analysis.

There are several ways of valuing a real option, including partial differential equations, dynamic programming, and Monte Carlo simulations (Dixit and Pindyck 1994, Trigeorgis 1996, Schwarz 2002). Given the incompatibility of the calculus of game theory and the stochastic calculus of real options, the use of Monte Carlo simulation was the chosen technique. In this technique, a random demand variable is integrated into the profit function for each firm:

$$\mathrm{p}_{At} = R(P_{At};\, P_{Bt}\text{¦}\, D_0;\, s) - C(Q_{At}(P_{At};\, P_{Bt})) \qquad (9)$$

$$\mathrm{p}_{Bt} = R(P_{At};\, P_{Bt}\text{¦}\, D_0;\, s) - C(Q_{Bt}(P_{At};\, P_{Bt})) \qquad (10)$$

where D_0 is the baseline level of demand (roughly proportional to the intercept of the demand curve) and s is the variability of annual demand. With the stochastic demand curve, Airbus' strategy is its optimal choice of entry date for each possible entry date by

$$\max_{t_A} \,?\, _{A\text{¦}\, tB=1}(tA) = \sum_{t=1}^{n} \frac{1}{(1+r)^t} \left[R(P^*_{At},\, P^*_{Bt},\, tA\text{¦}\, tB=1,\, D_o,\, s) - C\,(Q_{At}(P^*_{At},\, P^*_{Bt}))\right]$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$\max_{t_A} \text{?}_{A\text{¦}\, tB=n+1}(tA) = \sum_{t=1}^{n} \frac{1}{(1+r)^t} \left[R(P^*_{At}, P^*_{Bt}, tA\text{¦}\, tB=n+1, D_o, s) - C\,(Q_{At}(P^*_{At}, P^*_{Bt}))\right]$$

Boeing, and Boeing faces a similar problem. Given the stochastic specification of the demand curve, first a random series of annual demand is generated according to the specification of demand and then populate the pairs of payoffs for every permutation of entry dates in the (*n* + 1) x (*n* + 1) normal form entry game. The model then finds the Nash equilibrium pair of optimal entry dates for that particular realization of demand.

3.3 Cash Flow

To perform the simulations of the mathematical model of endogenous entry, a cash flow model is constructed for each firm, where cash flows depend on the entry timing and pricing decisions of both firms. First a derived demand model for aircraft is constructed that assigns market share to each aircraft according to its relative operating margin on a per seat basis for the airlines. In other words, demand is determined by the relative cash flow the aircraft generates for its airline customers after covering the allocated purchase price. Quantity demanded for a particular aircraft is simply determined by market share times total market demand. Since operating margins depend on price and on the efficiency of the aircraft being sold, revenue in a particular period changes depending on the entry decision of the firm. For example, if Airbus has not yet entered with the A380, Boeing is selling the 747 or the 747X as a monopolist. After Airbus' entry, quantity demanded will be determined by market shares depending on relative prices.

Each firm is allowed to enter at any time within a 20 year horizon. Uncertain market demand fluctuates over the time horizon according to the specification of the uncertainty (in this case demand follows a Markov process). For every realization of demand, both firms observe a series of annual levels of demand that determine cash flows that include revenues given from the demand curve, fixed costs determined by the capital investment, and variable costs determined by a learning curve. Each firm then chooses an entry date and sets prices for each period in the horizon to maximize its net present value (NPV).

With the cash flows, the study implements the integrated model of game-theory and real options as explained above. Airbus and Boeing are ultimately choosing their optimal entry date conditional on every possible the entry date of the other firm. The conditional payoffs from these decisions rely on optimal pricing decisions in the specific competitive environment each year (differentiated Bertrand or monopoly). The Nash equilibrium pair of entry decisions occurs where both firms are simultaneously choosing their best response to their rival. This gives the entry timing and payoffs for that particular realization of demand.

The cash flows for the model are constructed by building the annual profit functions for each firm given starting demand, relative efficiency of the aircraft, variable costs, and depreciated fixed costs (capital investment). In any given year where the competitive environment is a Bertrand duopoly, the model finds each firm's reaction function and solve for price. Of course, when Boeing is competing alone the model solves for the monopoly price. However, Airbus can never act as a monopolist. Even when Airbus is able to preempt Boeing with its superjumbo, Boeing is still allowed to sell its incumbent product, the 747.

Airbus and Boeing compete over a 44 year time horizon that allows each to enter as late as period 20, build the project, and fully depreciate its assets assuming 20-year, straight-line depreciation. At the end of the 44 year horizon, a terminal value for the project is constructed by assuming that the last cash flow will continue in perpetuity. The dynamics of the stochastic market demand is defined by a Markov process (random-walk) with a normal distribution around the demand of the previous period:

$$dD = s\ dz \qquad (11)$$

where *dD* is the change in the level of market demand and *dz* is an increment to a Gauss Wiener process with variance s. The random walk has a lower bound for demand of 0. This is just the obvious result of the fact that Demand cannot be negative. This actually transforms the data gathered into a form similar to that of a log-normal distribution. Thus the terminal value given by assuming the last cash flow will continue into perpetuity is actually a lower value than the true expected value since the median, $\tilde{x}$, and the mean, $\bar{x}$, of the population of all possible demand paths must always follow the inequality $\bar{x} \geq \tilde{x}$. The only time that $\bar{x}$ is equal to $\tilde{x}$ is when the variance is equal to 0. This fact should cancel out any worry that the cash flows will not actually continue on into infinity. Any discrepancies that may occur because of these two facts will not be of a magnitude to have any biasing of the results

Thus, the total demand expectation at period i, can be seen as a sum of independent random variables, Xi, with a starting value for the mean equal to current demand, where X is approximated by a normal distribution. Thus, the sum of Xi‘s can be shown to be normal with a variance of n. This shows that the variance grows linearly with time and thus the standard deviation grows as the $\sqrt{t}$.

3.4 The Demand Model

The construction of the demand model is crucial for the determination of cash flows, so a full description of it is included here.

Since there has never been an alternative to the Boeing 747 in the VLA segment of the aircraft industry, historical data can not be relied upon to estimate a demand curve after Airbus and Boeing launch their new aircraft. Instead, the model employs a derived demand model for the VLA segment that determines market demand and allocates market share to each aircraft based on its contribution to customer profitability. More specifically, market share is determined by the operating margin per seat that the aircraft delivers to airlines relative to the margins from other aircraft options. Demand for specific aircraft is then determined as the product of market share times total demand. To allow market demand to change with changes in prices (and provide slope to the demand curves), a “demand shift factor” is constructed that rescales total demand up or down depending on an industry composite margin given the various aircraft in the market, depending on the specific scenario being tested, relative to the margin earned during the prelaunch condition of Boeing 747 as a monopolist. In other words, the known demand for the Boeing 747 at historical prices is used as an anchor and allows total demand for jumbo aircraft to change as the composite contribution margin of the aircraft in the market changes when relative prices change.

To construct the demand curve, let the price per seat of an aircraft be given by P_{si} where i indicates the type of aircraft where each type has a given number of seats. Revenue per seat mile assumes an industry average ticket price per passenger and is therefore constant across aircraft, $R_{sm} = R = \$0.116$. Variable expense per seat mile begins with the industry average of \$0.06 and which is then rescaled by an operating expense factor for the particular aircraft. For example, the Airbus A380 is expected to incur only 80% of the variable costs per seat mile of a Boeing 747 due to the A380's superior design and resulting efficiency. The variable cost per seat mile, as influenced by the efficiency factor is $V_{smi} = V \cdot E_i = 0.06E_i$ where V_{smi} is variable cost per seat mile for aircraft i, V is the industry average variable cost, and E_i is the efficiency factor for each aircraft. In the notation, $E_{B7} = 1.0$ is the efficiency factor for the Boeing 747 and $E_A = 0.8$ is the efficiency factor for the A380. Efficiency of the Boeing 747X is denoted as E_{BX} and is allowed to vary between 0.8 and 1.0 in the analysis. Fixed cost per seat mile begins with the industry average fixed cost per seat mile (F = \$0.045 per seat mile) which is then rescaled according to the price per seat of the particular aircraft relative to the price per seat of a 747. This gives a fixed cost per seat mile for each aircraft of $F_{smi} = \mathrm{F}\cdot\mathrm{P}_{si} / P_{s0} = 0.045\cdot\mathrm{P}_{si}/\$0.36$ where P_{si} is the price per seat of aircraft i and $P_{so} = \$0.36$ is the price per seat of a 747 before any entry decisions by Airbus (pre-launch price per aircraft for a 747 is \$150M). Taking all of these conditions together, an airline's expected margin per seat for a particular aircraft is simply revenue per seat mile less variable and fixed costs per seat mile, $M_i = R - V - E_i - F_i = R - V - E_i - F\cdot P_{si} / P_{s0}$.

Total demand and market share for a specific aircraft are both ultimately determined by the relative margins earned by the airlines. First the market share for each firm is constructed based on relative customer margins. It is then assumed that each aircraft will sell according to the margin it contributes to customers as a fraction of the sum of the margins in the market. Thus market share for aircraft i is given by $MS_i = M_i / (M_A+M_B)$ where M_A and M_B are the margins of the A380 and Boeing's aircraft respectively. To get demand for each aircraft, market share must be applied to the total market demand. We can draw upon relative margins to determine the total market demand as a function of prices since customer margins depend on aircraft price. To allow total market demand to vary based on price, the market demand shift factor (DS) is constructed as a function of market shares which depend upon prices, for each of three situations:

$$DS = 0 \qquad \text{if } M_i = 0 \text{ for } i = A,B \tag{12}$$

$$DS = \left[\frac{\frac{M_A+M_B}{2}}{M_0}\right]^b \qquad \text{if } M_i > 0 \text{ for } i = A,B \tag{13}$$

$$DS = \left[\frac{\max(M_A,M_B)}{M_0}\right]^b \qquad \text{if } M_i > 0 \text{ and } M_j = 0 \tag{14}$$

where M_0 is the margin earned by airlines buying a 747 in the prelaunch stage. Incorporating the market demand shift factor, it is found that total market demand is the known market demand from the prelaunch stage (Q_0) times the demand shift factor, $\overline{Q} = Q_0 \cdot DS$ where $\overline{Q}$ is total market demand.

With these elements, the demand curve is constructed for each aircraft in each possible launch scenario. Beginning with simplest case: Airbus does not launch and Boeing does not launch (NL-NL). In this case, Boeing continues as a monopolist in the VLA segment producing the 747. As a monopolist, Boeing's individual demand is also the market demand. The demand curve is given as the premarket quantity times the demand shift factor as defined in equation (10) where, without an aircraft, Airbus delivers a margin of \$0:

$$Q_{B7} = \overline{Q} = Q_0 \left[\frac{M_{B7}}{M_0} \right]^b$$

$$= Q_0 \left[\frac{R - V - \frac{FP_{sB7}}{P_{s0}}}{M_0} \right]^b \tag{15}$$

The demand curve for the case where Airbus does not launch and Boeing does launch (NL-L) is similar because Boeing is still a monopolist:

$$Q_{BX} = \overline{Q} = Q_0 \left[\frac{M_{BX}}{M_0} \right]^b$$

$$= Q_0 \left[\frac{R - V \cdot E_X - \frac{FP_{sBX}}{P_{s0}}}{M_0} \right]^b \tag{16}$$

When Airbus launches and Boeing does not launch (L-NL), there is a duopoly structure with Airbus selling the A380 and Boeing selling the 747. In this case, market demand is determined using the demand shift factor in equation (9):

$$\overline{Q} = Q_0 \left[\frac{\frac{M_A + M_B}{2}}{M_0} \right]^b = Q_0 \left[\frac{\frac{2R - VE_A - V - \frac{FP_{sA} + FP_{sA}}{P_{s0}}}{2}}{M_0} \right]^b \qquad (17)$$

Firm demand for Airbus and Boeing is given by market share multiplied by market demand:

$$Q_A = \overline{Q} \cdot MS_A$$

$$= Q_0 \left[\frac{\frac{2R - V \cdot E_A - V - \frac{FP_{sA} + FP_{sB7}}{P_{s0}}}{2}}{M_0} \right]^b \left[\frac{R - VE_A - \frac{FP_{sA}}{P_{s0}}}{2R - VE_A - V - \frac{FP_{sA} + FP_{sB7}}{P_{s0}}} \right] \qquad (18)$$

$$Q_B = \overline{Q} \cdot MS_B$$

$$= Q_0 \left[\frac{\frac{2R - V \cdot E_A - V - \frac{FP_{sA} + FP_{sB7}}{P_{s0}}}{2}}{M_0} \right]^b \left[\frac{R - V - \frac{FP_{sB7}}{P_{s0}}}{2R - VE_A - V - \frac{FP_{sA} + FP_{sB7}}{P_{s0}}} \right] \qquad (19)$$

When both Airbus and Boeing launch (L-L), there is a duopoly structure with Airbus selling the A380 and Boeing selling the 747X. In this case, market demand is determined as before:

$$\overline{Q} = Q_0 \left[\frac{\frac{M_A + M_{BX}}{2}}{M_0} \right]^b = Q_0 \left[\frac{\frac{2R - VE_A - V_X - \frac{FP_{sA} + FP_{sBX}}{P_{s0}}}{2}}{M_0} \right]^b \qquad (20)$$

Firm demand for Airbus and Boeing is given by market share times market demand:

$$Q_A = \overline{Q} \cdot MS_A \qquad (21)$$

$$= Q_0 \left[\frac{\frac{2R - VE_A - VE_X - \frac{FP_{sA} + FP_{sB7}}{P_{s0}}}{2}}{M_0} \right]^b \left[\frac{R - VE_A - \frac{FP_{sA}}{P_{s0}}}{2R - VE_A - VE_X - \frac{FP_{sA} + FP_{sB7}}{P_{s0}}} \right]$$

$$Q_B = \overline{Q} \cdot MS_B \qquad (22)$$

$$= Q_0 \left[\frac{\frac{2R - VE_A - VE_X - \frac{FP_{sA} + FP_{sBX}}{P_{s0}}}{2}}{M_0} \right]^b \left[\frac{R - VE_X - \frac{FP_{sX}}{P_{s0}}}{2R - VE_A - VE_X - \frac{FP_{sA} + FP_{sBX}}{P_{s0}}} \right]$$

3.5 Assumptions

There are a variety of variables in the model of firm profits that are unknown. For example, we can estimate Boeing's discount rate as its weighted cost of capital, but Airbus is a consortium of companies with headquarters in Europe and does not report the data needed to compute its weighted cost of capital. Similarly, it is widely acknowledged that aircraft variable costs follow a learning curve but we would need internal cost and output data to estimate learning curves for earlier aircraft models. Of course, there is no guarantee that the new aircraft would follow the learning curves of prior models. Other relevant variables include the baseline level of demand measured as the units sold over 20 years, SG&A, capital investment of each firm, relative efficiency of each aircraft, and the corporate tax rate. To find information to improve the assumptions, information was collected from Airbus' and Boeing's websites and from trade journals. The initial assumptions are listed in table 3-2.

Table 3-2. Assumed parameters for simulation model

Variable	Value	Units	Source
Starting Demand (D0)	1000	Units/20 years	Ave of published projections
Efficiency – 747	100%	Percentage	Baseline value
Efficiency – 747X	100%	Percentage	Assumption
Efficiency – A380	80%	Percentage	Published Reports
Investment – A380 (CIA)	$10,000	Millions	Ave of published reports
Investment – 747X (CIB)	$4,000	Millions	Ave of published reports
Discount Rate – Airbus	14.0%	Percentage	Assumption
Discount Rate – Boeing	13.8%	Percentage	WACC
Learning Rate – 747X	82%	% Cost Reduction	Assumption
Learning Rate – A380	82%	% Cost Reduction	Assumption
Initial VC 747	$500	Millions	Assumed starting cost
Initial VC 747X	$500	Millions	Assumed starting cost
Initial VC A380	$500	Millions	Assumed starting cost
Corporate Tax Rate	35%	Percentage	Average US tax rate
Depreciation	5%	Annual %	20-yr straight line
SG&A	3%	% of Revenue	Assumption

3.6 Model Programming

The model was programmed into both Mathematica and C. Mathematica was used for the symbolic derivations, and C was used to computationally crunch the numbers of each realization of the model. Due to the high volume of realizations required for a proper Monte Carlo simulation the C program was compiled and run on the supercomputer Mary Lou.

CHAPTER 4. METHODOLGY OF ANALYSIS

4.1 Monte Carlo Simulation

After repeatedly generating realizations of the random demand variable and finding the Nash equilibrium entry dates, we take the expectation of the entry date and payoffs of the entry decision for the whole. By generating a large enough sample of such simulated entry dates and payoffs, we find a close approximation of the expected value of the real option to defer investment given the specification of demand.

In many cases, such as the starting variable cost of the 747, inside knowledge from the firms would ensure that the assumptions are correct. In many other cases, such as the starting variable cost of the 747X, even the firms do not know the correct assumptions a priori. For example, we start with the assumption that the cost of the A380 project will be $10B. However, Airbus acknowledges that the precise cost of the project is unknown and is expected to fall within the range of $7-15B.

We have already discussed the uncertainty surrounding demand and it is likely that the operating efficiency of the finished aircraft will vary from initial projections which will influence demand in turn. Since the optimal entry decisions will surely depend on the realization of these unknown parameters, it is natural for the decision-maker to test the sensitivity of entry dates and the option value of delay to changes in the critical assumptions of the model. This is done by varying the assumptions within relevant ranges and determining the optimal entry timing in the integrated game-theoretic real-options model for every permutation. This affords the decision-maker a view of the impact of the unknown variables on the optimal decision. This sensitivity analysis also allows us to test the hypotheses regarding how these variables influence equilibrium outcomes and the option value of delay.

A total of 900 runs of 1,000 realizations each were run creating a total of 900,000 data points for statistical analysis. Table 4-1 shows the variables changed for these runs and the levels for each. The 900 runs come from the permutations of all these levels.

Table 4-1. Variable matrix

s	D0	CIB	CIA	Efficiency 747X
0.05	500	$2 B	$6 B	100%
0.1	700	$3 B	$8 B	0.8
0.15	900	$4 B	$10 B	
0.2	1100		$12 B	
0.25	1300		$14 B	
	1400			

The Monte Carlo simulation appropriately integrates the game theory and real option components of the decision, and the data produced are valuable and can be analyzed at this point in a myriad of different ways.

4.2 The Hazard Model

A hazard model is used to find the probabilities of entry for a firm. A hazard model is just the conditional probability of an event happening, given it hasn't already occurred. In this case it is the probability of entry given that the firm has not already entered. Hazard analysis has been used to evaluate investment decisions in the pharmaceutical industry by looking at the time to a second patent. (McGrath 2004) This study uses hazard rates in a similar way looking at the time to investment being the time to launch the new product.

A hazard rate is the conditional probability of exercising an option (Kogut 2004), so using the hazard function gives a way of evaluating the option from a manager's decision of do I launch today given the level of demand he sees, the variance in his demand forecast, etc.

A Hazard function also incorporates information on both censored and uncensored cases, i.e., whether or not a launch decision is made. This allows information to be gathered by those cases that no decision is made, since a "no" decision is as much a decision as a "yes" decision. This insures that we are using all of the information available increasing the validity, and accuracy of the model.

The intuitive definition of a Hazard is that is just the PDF divided by one minus the CDF:

$$\frac{f(x)}{1-F(x)} \tag{23}$$

However, the more accurate, definition is: If *T* is the duration since the first opportunity to launch occurred, then the instantaneous (hazard) rate of the firm launching at time *t* is defined as

$$h(t) = \lim_{\Delta t \to o} \frac{\Pr\{t \le T < t + \Delta t \mid T \ge t\}}{\Delta t} \qquad (24)$$

In using this definition hazard rates are not probabilities. Because of their uniqueness, Allison (1995) points out that there are three points have to be understood in order to properly interpret any given hazard function and its parameters. They are:

1. Even though it is helpful to think of a hazard as an instantaneous probability, it is not a true probability and its' hazard can be greater than 1.0.
2. Because a hazard is defined in terms of probability, it is itself an unobserved quantity. In other words we may only estimate the hazard.
3. Hazards are characteristics of individuals, not populations or samples. Each individual will have a different hazard given its' conditions and current state.

Since a hazard is not a probability, the way of interpreting it is as a rate, or the number of occurrences per interval of time. So in our case, a hazard of .6 would mean that in that given year a launch will occur .6 times. If the rate is 1.4, a launch will occur 1.4 times. This may be a little confusing in our situation, but think of the hazard to catch a cold. It may easily be 3.5 for a time period of a year. However, that does not mean that it is guaranteed that you will catch a cold, and you may catch more than 3.5 over the year.

In the case of a product launch, the event can only occur once, but the chances increase with the hazard. In general, a .63 hazard can be interpreted to be a 63% chance of the event, but it is still necessary to understand where that comes from and why the hazard may be greater than 1.0.

The hazard rate is modeled using semiparametric Cox models (Cox, 1972; Kalbfleisch and Prentice, 1980; Allison, 1995). The equation that we estimate takes the following specification:

$$h_i(t) = ?_0(t)exp\{ß_1X_{i1} + ... + ß_kX_{ik}\} \tag{25}$$

where h(t) is the hazard rate of the launch event. ?(t) is an unspecified baseline rate for the transition. Xik’s are time-constant covariates and the ßk’s are unknown regression parameters. Because ?(t) is an unspecified step function, the Cox model is an extremely flexible method for modeling time dependence (Sørensen and Stuart, 2000).

CHAPTER 5. ANALYSIS AND RESULTS

Once the Monte Carlo simulation has generated the data, a statistical software package can be used to perform the Cox Regression. Almost all modern statistical software packages offer a hazard regression option that includes a Cox Regression. For the purpose of this study SPSS was the statistical software package chosen to analyze the data produced by the Monte Carlo simulation.

5.1 Using the Hazard Model to Make Real Time Decisions

The Hazard model approach can accurately show the influence that different variables have on the decision either separately or with respect to each other.

However, in the use of a hazard model to either regress or to a model decision making there is one major assumption that needs to be made that either current real options literature is unaware of or chooses to ignore. This assumption is that the firm making the decision always makes the right decision. The reason that this assumption is necessary can be shown by comparing the way the model evaluates a decision as opposed to the classical use of a hazard model to regress and predict the probability of death. In the first period of the model the scenarios both perform in the same way. Unfortunately, after the first period ends the two models diverge in what is being measured and/or predicted.

This problem only occurs in real time decision making. The hazard model can regress another model such as the one built for this study since the right decision is always made, or in other words just as in death the outcome is not based on a decision, but an actual outcome. Unfortunately the second that you try to apply the model to predict actual decision making behavior the assumption of absoluteness of a correct decision is violated.

Thus, rather than develop a method to evaluate real options in real time using a hazard model this study has shown that the hazard model has been inappropriately used in recent strategy and decision analysis research to model and evaluate real decisions. For example, the McGrath study of time until a second patent in the pharmaceutical industry assumes that not only was the correct decision made in the filing for the first patent, but that all related decisions in between the two patents also satisfy the assumption of optimality. While this does not eliminate all value of hazard analysis for evaluating real options, it does force an assumption that calls any results in to serious question. Until another tool is developed real options researches may have to settle for a suboptimal way to evaluate real options, but since the error introduced is independent and unique for every study results will have to be taken at face value and study comparisons avoided.

5.2 *Real Time Decision Making*

Fortunately this study has developed a method to evaluate both real options and game theory in the forward looking world of predictive models. This is possible without using a hazard model since the original model that this study has developed endogenously captures the real option value to delay as well as game theoretical dynamics. In fact, in losing the ability to use a hazard model, the only real loss is in the ease that a decision can be reached. In reality, by not fitting the original data to another model but instead basing the decision off of the original data, no information is lost from the data resulting in a model with more accurate predictive power. A decision maker can go directly to the model to evaluate different decisions.

While many less sophisticated managers may not feel comfortable using and interpreting the results of the model directly, it is not unusual that for highly important strategic decisions, such as important new product launches, many different divisions within a company are sought out for their input, and in most major corporations analysis at a similar level of complexity is routinely performed by either an internal or external consulting team or captured into a software package. Complicated forecasting is already performed in almost all situations and this type of analysis is a natural extension of forecasting that should be welcomed by the "Geek Groups" that seem to inhabit this domain.

5.2.1 Testing hypothesis

Hazard regression has the ability to regress over multiple sets of data and give time dependency to the decision, but can also be done as a straight regression with no other variables other than event time and give you the baseline. This flexibility allows a modeler the ability to run various scenarios without having to worry about the underlying methods feasibility which could possibly require changing the regression format.

The equation will not predict the optimal time to launch. It will however help a manager make his decision in whether or not he should launch in a real time decision. For example, an Airbus manager can plug into the equation that Boeing has or has not already launched the 747X. However, what constitutes credible commitment on Boeing's part is up to interpretation. This is further complicated since it can be shown that Boeing prefers to delay the launch of Airbus even if it means that they do not launch because of their current monopolist position in the market. Thus, a manger for Airbus would need to evaluate the model under both situations, determining what his choice would be given the two scenarios, and then make a judgment on what the best course to follow is given the circumstances.

Following this logical process, a manager can test hypotheses that he has with regard to whether and to what degree a variable influences the firms decision to launch or not. A manger can even look at what a competitor's action would best suit them and then try to "change" or distort what they see to try and create that scenario.

Table 5-1 is a Survival table for Boeing under all conditions run in the simulation. In the table the baseline of the Hazard Model is shown, and well as the results when plugging in the mean of the different variables into the equation regressed. Table 5-2 shows the different variables regressed and their respective significance and regression equation coefficients, and the 95% CI for each. As you can see all variables selected for this regression were statistically significant at the most stringent of confidence levels. In fact the significance for many of them was so high that the value of significance was essentially 0, signifying a 100% confidence of significance.

Tables 5-3, 5-4 are the same tables as shown for Boeing but now run for Airbus. If you look at the corresponding variables you can see how the same variable can have either the same or opposing effect on the two firms. For example, the variable of efficiency, which represents the efficiency for the 747X, pushes Boeing to an earlier launch, but inhibits Airbus from launching under the general parameters regressed. On the other hand, demand (launchBD or launchAD) which has the same input to both pushes both towards a launch decision.

Using Hazard Regression in this way a manager can see what factors effect his decision and how they change under the different scenarios that he wishes to evaluate.

Table 5-1. Boeing survival table

Time	Baseline Cum Hazard	At mean of covariates		
		Survival	SE	Cum Hazard
1	0.708594573	0.715715	0.000304	0.334473472
2	0.823535151	0.677919	0.000381	0.388728155
3	0.903764006	0.652726	0.000425	0.426598081
4	0.957159053	0.63648	0.00045	0.451801812
5	1.026654477	0.61594	0.000479	0.484605303
6	1.088723788	0.598156	0.000502	0.513903492
7	1.147454964	0.581801	0.00052	0.541626002
8	1.204152397	0.566437	0.000537	0.568388537
9	1.252379472	0.553689	0.000549	0.591152863
10	1.320428459	0.536186	0.000564	0.623273601
11	1.383713316	0.520406	0.000577	0.653145557
12	1.442969814	0.506052	0.000587	0.681116032
13	1.504245027	0.491625	0.000596	0.710039388
14	1.567947125	0.477062	0.000605	0.740108292
15	1.635587919	0.462071	0.000612	0.772036353
16	1.698125005	0.448631	0.000618	0.801555344
17	1.769279973	0.433813	0.000623	0.835142179
18	1.8650879	0.414631	0.000629	0.880365797
19	2.045820746	0.380726	0.000632	0.965675993
20	3.473041662	0.194105	0.000475	1.639358171

Table 5-2. Variables in the equation

	B	SE	Sig.	Exp(B)	95.0% CI for Exp(B)	
					Lower	Upper
Eff	0.521328	0.015289	0	1.684263	1.634541	1.735498
StDev	0.004437	0.000189	0	1.004447	1.004074	1.004819
CIB	-0.00029	1.67E-06	0	0.99971	0.999707	0.999713
AbB	-3.09169	0.004791	0	0.045425	0.045001	0.045854
launchBD	0.025491	6.19E-05	0	1.025818	1.025694	1.025943

Table 5-3. Airbus survival table

Time	Baseline Cum Hazard	At mean of covariates		
		Survival	SE	Cum Hazard
1	1.415859935	0.773891	0.000273	0.256324805
2	1.569093785	0.752717	0.000335	0.284065993
3	1.704650833	0.734469	0.000382	0.308607004
4	1.808000319	0.720855	0.000415	0.327317214
5	1.949789307	0.702587	0.000454	0.352986444
6	2.096149656	0.684215	0.000491	0.379483266
7	2.220295631	0.669009	0.000519	0.401958436
8	2.355328482	0.652852	0.000546	0.426404547
9	2.474735714	0.638891	0.000569	0.448021823
10	2.636902207	0.620407	0.000595	0.477380161
11	2.809171879	0.601356	0.000621	0.508567561
12	2.979067748	0.583142	0.000642	0.539325212
13	3.149059417	0.565469	0.000662	0.570100206
14	3.331346042	0.547112	0.00068	0.603101058
15	3.495819305	0.531062	0.000694	0.63287701
16	3.683879749	0.513285	0.000708	0.666923144
17	3.871430735	0.49615	0.00072	0.700877046
18	4.091854431	0.476741	0.000732	0.740782166
19	4.387089731	0.451929	0.000744	0.794231048
20	5.550849898	0.366075	0.000737	1.004916152

Table 5-4. Variables in the equation

	B	SE	Sig.	Exp(B)	95.0% CI for Exp(B)	
					Lower	Upper
Eff	-1.2152488	0.0171	0	0.296636	0.286859	0.306747
StDev	0.00707311	0.000206	0	1.007098	1.006691	1.007505
CIA	-4.7249E-05	5.68E-07	0	0.999953	0.999952	0.999954
launchAD	0.02883713	7.53E-05	0	1.029257	1.029105	1.029409
BbA	-2.68008978	0.005049	0	0.068557	0.067882	0.069239

5.2.2 Risk management

Another extremely valuable advantage of using this model is its' ability to not only show what the percent of time a launch is a correct decision, but a manger can look at individual paths and find out what happens in the non-optimal realizations. If the decisions are non-optimal by only a small delta in NPV, he is not concerned about the downside of his decision. Conversely, if significant losses occur in the non-optimal solutions, he can evaluate what the risk is, and make a decision that reflects both his and his firm's adverseness to risk. For example, risk may be tied closely to a competitor's response, or solely to fluctuations in demand. A manager can then evaluate where the risk comes from and take educated, calculated risks.

5.2.3 Crosstabulation

Another powerful way for a manager to interpret the data is to use scenario analysis and crosstabs to evaluate specific situations and what the best response would be given your decision criteria. Underlying dynamics that a single regression cannot find can also be uncovered by looking at outcomes of the Monte Carlo simulation under different scenarios. An example of this would be to look at the behavior of Boeing under different demand scenarios cross tabulated with the Airbuses expected capital investment(CIA). Table 4-5 shows part of the table that would be created. Since the table would have over 400 rows only a few of the rows that demonstrate the unexpected behavior that this crosstabulation exposes are shown. The entire table can be found in the appendix.

Table 5-5. BL * Do * CIA Crosstabulation

CIA	Yr		Do					
			500	700	900	1100	1300	1500
6000	1	Count	8714	7968	5340	3582	3443	4473
		Expected Count	5586	5586	5586	5587	5587	5587
		% within Do	29.05	26.56	17.80	11.94	11.47	14.91
14000	1	Count	14429	18980	18598	16751	15760	14086
		Expected Count	16620	16620	16619	16617	16618	15509
		% within Do	48.09	63.26	61.99	55.84	52.54	50.31

One would expect that as demand increases Boeing would be more likely to launch. However, looking at Boeing’s behavior the opposite occurs. To understand this it is important to first remember that under most conditions Boeing would prefer to maintain its monopolistic position if at all possible. This is what drives this strange behavior, for if Boeing can pre-empt Airbus and keep them out of the market they will do so even at increases risk. On the other hand, Boeing prefers to delay their launch until after Airbus’s if this is not possible.

This incentive to maintain monopolistic conditions drive the results of why when demand is only 500 Boeing is more likely to enter in the first period then when demand is triple that at 1500. This occurs because Airbus will never enter into the market that has Do at 500 if it has to compete against Boeing to do so. Thus, when Boeing preempts Airbus at low demand they are almost completely assured of maintaining their monopolistic position. The chances of Boeing preempting and keeping Airbus out of the market decrease with higher demand. This is represented by Boeing's increasing reluctance to launch as demand increases. This reverses slightly once demand reaches 1500 since at that point the high level of demand starts to outweigh the loss of an Airbus entry, and since Airbuses decision becomes increasingly independent of Boeing's decision. Also, the value of waiting is demonstrated in all cases by lower than expected launch rates.

A similar argument follows for when CIA is 1400, except here the factor limiting Airbus from launching is its high investment cost. Thus, Boeing can easily preempt, shown by more frequent launches. At the same time the importance of demand in keeping Airbus out is diminished so Boeing reacts more as expected to demand, where its probability of launching is positively related with demand. This effect changes however as demand increases beyond a point where CIA becomes decreasingly strong as an incentive to not launch.

CHAPTER 6. CONCLUSIONS AND FUTURE RESEARCH

6.1 Conclusions

While high level quantitative tools have begun to be used to evaluate corporate strategy, these tools are still mainly confined to research groups within large corporations. Both real options and game theory have been evaluated and used by these groups. However, they have not been adequately integrated together in the academic world, let alone in industry. This thesis help bridge the gap between strategic decision making, and the theoretical world of economic decision analysis creating a prescriptive model companies can use to evaluate strategically important new product launches.

To bridge this gap a method that is able to handle the integration of game-theoretic and options-theoretic reasoning to the strategic analysis of new product introduction is developed. Not only was a method developed that could incorporate the two methods it was done in a way that is accessible and useful outside of the academic world.

In developing this methodology the fallacy of using a hazard model to evaluate decision making such a real options approach was discovered. Stricter and more unrealistic assumptions are required to use hazard regression than has previously been acknowledged. This thesis not only discovered these assumptions but also explained their limitations and when it hazard regression can still be a useful tool.

The study also demonstrates how optimal behavior is often counterintuitive when decisions are influenced by competition and high uncertainty. This leads to poor decisions unless a quantitative model can show how different variables interact and the economic environment that this creates. For example, Boeing is more likely to launch the 747X when demand is extremely low, than when demand is much higher. This is because of the ability Boeing possesses to preempt their competition and keep them out of the market. However, as uncertainty rises, the incentive to wait becomes stronger and it becomes almost impossible to separate the two competing forces.

This study finally offers a prescriptive model that can aid in making important strategic decisions with respect to new product introductions. In short, a practical method is developed that can be used by managers that are drowning in a sea of uncertainty.

6.2 Future Research

Using this model as a baseline future research can now look at some of the more intricate details of such as learning from events and the effect these events have on real options thinking. Learning can occur in many ways. For example, a firm may wait until another firm launches their own product to see how the market receives it. The model developed could be expanded to include such factors, and also possibly used to measure empirically whether or not firms currently use real options and game-theoretic thinking in their decisions.

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APPENDIX

BL * Do * CIA Crosstabulation

CIA			Do						Total
			500	700	900	1100	1300	1500	
6000 BL	1	Count	8714	7968	5340	3582	3443	4473	33520
		Expected Count	5586.2	5586.4	5586.4	5586.9	5586.9	5587.3	33520
		% within BL	25.996	23.771	15.931	10.686	10.271	13.344	100
		% within Do	29.054	26.566	17.804	11.942	11.478	14.911	18.62553
	2	Count	583	358	266	385	617	956	3165
		Expected Count	527.45	527.47	527.47	527.52	527.52	527.56	3165
		% within BL	18.42	11.311	8.4044	12.164	19.494	30.205	100
		% within Do	1.9439	1.1936	0.8869	1.2835	2.0569	3.1869	1.758646
	3	Count	359	210	182	278	405	629	2063
		Expected Count	343.8	343.81	343.81	343.85	343.85	343.87	2063
		% within BL	17.402	10.179	8.8221	13.476	19.632	30.49	100
		% within Do	1.197	0.7002	0.6068	0.9268	1.3502	2.0968	1.146315
	4	Count	182	111	124	189	334	501	1441
		Expected Count	240.15	240.15	240.15	240.18	240.18	240.19	1441
		% within BL	12.63	7.703	8.6051	13.116	23.178	34.768	100
		% within Do	0.6068	0.3701	0.4134	0.6301	1.1135	1.6701	0.800698
	5	Count	215	151	144	263	404	551	1728
		Expected Count	287.97	287.98	287.98	288.01	288.01	288.03	1728
		% within BL	12.442	8.7384	8.3333	15.22	23.38	31.887	100
		% within Do	0.7169	0.5035	0.4801	0.8768	1.3468	1.8368	0.960171
	6	Count	189	104	118	237	384	475	1507
		Expected Count	251.14	251.15	251.15	251.18	251.18	251.19	1507
		% within BL	12.541	6.9011	7.8301	15.727	25.481	31.52	100
		% within Do	0.6302	0.3467	0.3934	0.7901	1.2802	1.5834	0.837371
	7	Count	132	78	150	251	368	434	1413
		Expected Count	235.48	235.49	235.49	235.51	235.51	235.53	1413
		% within BL	9.3418	5.5202	10.616	17.764	26.044	30.715	100
		% within Do	0.4401	0.2601	0.5001	0.8368	1.2268	1.4468	0.78514
	8	Count	140	97	147	235	326	390	1335
		Expected Count	222.48	222.49	222.49	222.51	222.51	222.52	1335
		% within BL	10.487	7.2659	11.011	17.603	24.419	29.213	100
		% within Do	0.4668	0.3234	0.4901	0.7834	1.0868	1.3001	0.741799
	9	Count	110	64	116	209	269	348	1116
		Expected Count	185.98	185.99	185.99	186.01	186.01	186.02	1116
		% within BL	9.8566	5.7348	10.394	18.728	24.104	31.183	100
		% within Do	0.3668	0.2134	0.3868	0.6968	0.8968	1.1601	0.62011
	10	Count	160	96	162	264	349	430	1461
		Expected Count	243.48	243.49	243.49	243.51	243.51	243.53	1461

	% within BL	10.951	6.5708	11.088	18.07	23.888	29.432	100
	% within Do	0.5335	0.3201	0.5401	0.8801	1.1635	1.4334	0.811811
11	Count	138	117	161	222	310	394	1342
	Expected Count	223.65	223.65	223.65	223.68	223.68	223.69	1342
	% within BL	10.283	8.7183	11.997	16.542	23.1	29.359	100
	% within Do	0.4601	0.3901	0.5368	0.7401	1.0335	1.3134	0.745688
12	Count	130	73	138	221	327	358	1247
	Expected Count	207.81	207.82	207.82	207.84	207.84	207.86	1247
	% within BL	10.425	5.854	11.067	17.723	26.223	28.709	100
	% within Do	0.4334	0.2434	0.4601	0.7368	1.0901	1.1934	0.692901
13	Count	106	96	175	258	309	345	1289
	Expected Count	214.81	214.82	214.82	214.84	214.84	214.86	1289
	% within BL	8.2234	7.4476	13.576	20.016	23.972	26.765	100
	% within Do	0.3534	0.3201	0.5835	0.8601	1.0301	1.1501	0.716238
14	Count	138	108	155	270	325	363	1359
	Expected Count	226.48	226.49	226.49	226.51	226.51	226.53	1359
	% within BL	10.155	7.947	11.405	19.868	23.915	26.711	100
	% within Do	0.4601	0.3601	0.5168	0.9001	1.0835	1.2101	0.755134
15	Count	133	132	191	239	277	284	1256
	Expected Count	209.31	209.32	209.32	209.34	209.34	209.36	1256
	% within BL	10.589	10.51	15.207	19.029	22.054	22.611	100
	% within Do	0.4435	0.4401	0.6368	0.7968	0.9235	0.9467	0.697902
16	Count	143	105	150	209	303	354	1264
	Expected Count	210.65	210.65	210.65	210.68	210.68	210.69	1264
	% within BL	11.313	8.307	11.867	16.535	23.972	28.006	100
	% within Do	0.4768	0.3501	0.5001	0.6968	1.0101	1.1801	0.702347
17	Count	146	119	178	235	307	319	1304
	Expected Count	217.31	217.32	217.32	217.34	217.34	217.36	1304
	% within BL	11.196	9.1258	13.65	18.021	23.543	24.463	100
	% within Do	0.4868	0.3968	0.5935	0.7834	1.0235	1.0634	0.724573
18	Count	216	167	245	322	373	405	1728
	Expected Count	287.97	287.98	287.98	288.01	288.01	288.03	1728
	% within BL	12.5	9.6644	14.178	18.634	21.586	23.438	100
	% within Do	0.7202	0.5568	0.8169	1.0735	1.2435	1.3501	0.960171
19	Count	468	335	390	520	604	694	3011
	Expected Count	501.79	501.81	501.81	501.86	501.86	501.89	3011
	% within BL	15.543	11.126	12.953	17.27	20.06	23.049	100
	% within Do	1.5604	1.1169	1.3003	1.7336	2.0136	2.3135	1.673075
20	Count	2746	2149	2773	3400	3982	4041	19091
	Expected Count	3181.6	3181.7	3181.7	3182	3182	3182.2	19091
	% within BL	14.384	11.257	14.525	17.809	20.858	21.167	100
	% within Do	9.1558	7.165	9.2455	11.335	13.275	13.471	10.608
21	Count	14844	17355	18688	18207	15980	13254	98328
	Expected Count	16387	16387	16387	16389	16389	16390	98328
	% within BL	15.096	17.65	19.006	18.517	16.252	13.479	100

		% within Do	49.493	57.864	62.308	60.698	53.274	44.183	54.63638
Total		Count	29992	29993	29993	29996	29996	29998	179968
		Expected Count	29992	29993	29993	29996	29996	29998	179968
		% within BL	16.665	16.666	16.666	16.667	16.667	16.669	100
		% within Do	100	100	100	100	100	100	100
8000 BL	1	Count	11434	11559	9729	7535	6051	6169	52477
		Expected Count	8746.9	8746.3	8746.9	8745.4	8745.1	8746.3	52477
		% within BL	21.789	22.027	18.54	14.359	11.531	11.756	100
		% within Do	38.113	38.533	32.43	25.121	20.174	20.565	29.15632
	2	Count	808	533	446	487	684	1019	3977
		Expected Count	662.89	662.84	662.89	662.78	662.76	662.84	3977
		% within BL	20.317	13.402	11.214	12.245	17.199	25.622	100
		% within Do	2.6933	1.7768	1.4867	1.6236	2.2805	3.3969	2.209629
	3	Count	498	314	267	343	438	636	2496
		Expected Count	416.03	416.01	416.03	415.97	415.95	416.01	2496
		% within BL	19.952	12.58	10.697	13.742	17.548	25.481	100
		% within Do	1.66	1.0467	0.89	1.1435	1.4603	2.1201	1.386782
	4	Count	274	169	167	202	346	510	1668
		Expected Count	278.02	278	278.02	277.98	277.97	278	1668
		% within BL	16.427	10.132	10.012	12.11	20.743	30.576	100
		% within Do	0.9133	0.5634	0.5567	0.6734	1.1536	1.7001	0.926744
	5	Count	324	218	206	296	437	540	2021
		Expected Count	336.86	336.84	336.86	336.81	336.79	336.84	2021
		% within BL	16.032	10.787	10.193	14.646	21.623	26.719	100
		% within Do	1.08	0.7267	0.6867	0.9868	1.457	1.8001	1.122871
	6	Count	277	164	149	263	380	483	1716
		Expected Count	286.02	286	286.02	285.98	285.97	286	1716
		% within BL	16.142	9.5571	8.683	15.326	22.145	28.147	100
		% within Do	0.9233	0.5467	0.4967	0.8768	1.2669	1.6101	0.953413
	7	Count	206	109	182	259	372	440	1568
		Expected Count	261.36	261.34	261.36	261.31	261.3	261.34	1568
		% within BL	13.138	6.9515	11.607	16.518	23.724	28.061	100
		% within Do	0.6867	0.3634	0.6067	0.8635	1.2402	1.4668	0.871184
	8	Count	216	145	167	231	323	390	1472
		Expected Count	245.35	245.34	245.35	245.31	245.3	245.34	1472
		% within BL	14.674	9.8505	11.345	15.693	21.943	26.495	100
		% within Do	0.72	0.4834	0.5567	0.7701	1.0769	1.3001	0.817846
	9	Count	165	92	128	207	266	353	1211
		Expected Count	201.85	201.84	201.85	201.82	201.81	201.84	1211
		% within BL	13.625	7.597	10.57	17.093	21.965	29.149	100
		% within Do	0.55	0.3067	0.4267	0.6901	0.8868	1.1767	0.672834
	10	Count	244	152	189	272	350	429	1636
		Expected Count	272.69	272.67	272.69	272.64	272.63	272.67	1636
		% within BL	14.914	9.291	11.553	16.626	21.394	26.222	100

	% within Do	0.8133	0.5067	0.63	0.9068	1.1669	1.4301	0.908965
11	Count	208	149	172	234	302	372	1437
	Expected Count	239.52	239.5	239.52	239.48	239.47	239.5	1437
	% within BL	14.475	10.369	11.969	16.284	21.016	25.887	100
	% within Do	0.6933	0.4967	0.5733	0.7801	1.0069	1.2401	0.7984
12	Count	194	85	137	224	321	346	1307
	Expected Count	217.85	217.84	217.85	217.82	217.81	217.84	1307
	% within BL	14.843	6.5034	10.482	17.138	24.56	26.473	100
	% within Do	0.6467	0.2834	0.4567	0.7468	1.0702	1.1534	0.726172
13	Count	156	112	175	259	308	335	1345
	Expected Count	224.19	224.17	224.19	224.15	224.14	224.17	1345
	% within BL	11.599	8.3271	13.011	19.257	22.9	24.907	100
	% within Do	0.52	0.3734	0.5833	0.8635	1.0269	1.1167	0.747284
14	Count	186	130	151	255	307	339	1368
	Expected Count	228.02	228	228.02	227.98	227.97	228	1368
	% within BL	13.596	9.5029	11.038	18.64	22.442	24.781	100
	% within Do	0.62	0.4334	0.5033	0.8501	1.0235	1.1301	0.760063
15	Count	179	158	188	236	265	285	1311
	Expected Count	218.52	218.5	218.52	218.48	218.47	218.5	1311
	% within BL	13.654	12.052	14.34	18.002	20.214	21.739	100
	% within Do	0.5967	0.5267	0.6267	0.7868	0.8835	0.9501	0.728394
16	Count	186	106	139	200	273	336	1240
	Expected Count	206.68	206.67	206.68	206.65	206.64	206.67	1240
	% within BL	15	8.5484	11.21	16.129	22.016	27.097	100
	% within Do	0.62	0.3534	0.4633	0.6668	0.9102	1.1201	0.688946
17	Count	190	131	178	227	307	315	1348
	Expected Count	224.69	224.67	224.69	224.65	224.64	224.67	1348
	% within BL	14.095	9.7181	13.205	16.84	22.774	23.368	100
	% within Do	0.6333	0.4367	0.5933	0.7568	1.0235	1.0501	0.748951
18	Count	263	179	224	315	362	388	1731
	Expected Count	288.52	288.5	288.52	288.48	288.47	288.5	1731
	% within BL	15.194	10.341	12.94	18.198	20.913	22.415	100
	% within Do	0.8767	0.5967	0.7467	1.0502	1.2069	1.2934	0.961747
19	Count	540	361	386	512	557	646	3002
	Expected Count	500.38	500.34	500.38	500.29	500.27	500.34	3002
	% within BL	17.988	12.025	12.858	17.055	18.554	21.519	100
	% within Do	1.8	1.2034	1.2867	1.707	1.857	2.1535	1.667917
20	Count	2901	1962	2386	2920	3536	3724	17429
	Expected Count	2905.1	2904.9	2905.1	2904.6	2904.5	2904.9	17429

			% within BL	16.645	11.257	13.69	16.754	20.288	21.367	100
			% within Do	9.67	6.5404	7.9533	9.735	11.789	12.414	9.683585
		21	Count	10551	13170	14234	14518	13809	11943	78225
			Expected Count	13039	13038	13039	13036	13036	13038	78225
			% within BL	13.488	16.836	18.196	18.559	17.653	15.267	100
			% within Do	35.17	43.903	47.447	48.401	46.039	39.813	43.46196
	Total		Count	30000	29998	30000	29995	29994	29998	179985
			Expected Count	30000	29998	30000	29995	29994	29998	179985
			% within BL	16.668	16.667	16.668	16.665	16.665	16.667	100
			% within Do	100	100	100	100	100	100	100
10000	BL	1	Count	12903	14942	12976	11658	10059	9131	71669
			Expected Count	11946	11946	11946	11944	11944	11944	71669
			% within BL	18.004	20.849	18.105	16.266	14.035	12.741	100
			% within Do	43.01	49.807	43.253	38.866	33.534	30.441	39.81899
		2	Count	933	666	562	587	748	1046	4542
			Expected Count	757.05	757.05	757.05	756.93	756.95	756.95	4542
			% within BL	20.542	14.663	12.373	12.924	16.469	23.03	100
			% within Do	3.11	2.22	1.8733	1.957	2.4937	3.4871	2.523516
		3	Count	592	384	328	386	457	628	2775
			Expected Count	462.53	462.53	462.53	462.46	462.47	462.47	2775
			% within BL	21.333	13.838	11.82	13.91	16.468	22.631	100
			% within Do	1.9733	1.28	1.0933	1.2869	1.5235	2.0936	1.541778
		4	Count	307	202	207	209	322	513	1760
			Expected Count	293.35	293.35	293.35	293.31	293.32	293.32	1760
			% within BL	17.443	11.477	11.761	11.875	18.295	29.148	100
			% within Do	1.0233	0.6733	0.69	0.6968	1.0735	1.7102	0.977848
		5	Count	369	270	233	310	429	535	2146
			Expected Count	357.69	357.69	357.69	357.63	357.64	357.64	2146
			% within BL	17.195	12.582	10.857	14.445	19.991	24.93	100
			% within Do	1.23	0.9	0.7767	1.0335	1.4302	1.7836	1.192308
		6	Count	311	203	190	258	362	456	1780
			Expected Count	296.69	296.69	296.69	296.64	296.65	296.65	1780
			% within BL	17.472	11.404	10.674	14.494	20.337	25.618	100
			% within Do	1.0367	0.6767	0.6333	0.8601	1.2068	1.5202	0.98896
		7	Count	237	126	190	253	352	439	1597
			Expected Count	266.19	266.19	266.19	266.14	266.15	266.15	1597
			% within BL	14.84	7.8898	11.897	15.842	22.041	27.489	100
			% within Do	0.79	0.42	0.6333	0.8435	1.1735	1.4635	0.887286
		8	Count	262	163	177	230	299	343	1474
			Expected Count	245.68	245.68	245.68	245.64	245.65	245.65	1474
			% within BL	17.775	11.058	12.008	15.604	20.285	23.27	100
			% within Do	0.8733	0.5433	0.59	0.7668	0.9968	1.1435	0.818948
		9	Count	194	120	134	191	237	330	1206
			Expected Count	201.01	201.01	201.01	200.98	200.99	200.99	1206
			% within BL	16.086	9.9502	11.111	15.837	19.652	27.363	100

	% within Do	0.6467	0.4	0.4467	0.6368	0.7901	1.1001	0.670048
10	Count	278	182	208	257	330	405	1660
	Expected Count	276.69	276.69	276.69	276.64	276.65	276.65	1660
	% within BL	16.747	10.964	12.53	15.482	19.88	24.398	100
	% within Do	0.9267	0.6067	0.6933	0.8568	1.1001	1.3502	0.922289
11	Count	234	175	176	231	292	373	1481
	Expected Count	246.85	246.85	246.85	246.81	246.82	246.82	1481
	% within BL	15.8	11.816	11.884	15.598	19.716	25.186	100
	% within Do	0.78	0.5833	0.5867	0.7701	0.9735	1.2435	0.822837
12	Count	233	126	131	183	314	326	1313
	Expected Count	218.85	218.85	218.85	218.81	218.82	218.82	1313
	% within BL	17.746	9.5963	9.9772	13.938	23.915	24.829	100
	% within Do	0.7767	0.42	0.4367	0.6101	1.0468	1.0868	0.729497
13	Count	178	126	154	242	295	312	1307
	Expected Count	217.85	217.85	217.85	217.81	217.82	217.82	1307
	% within BL	13.619	9.6404	11.783	18.516	22.571	23.871	100
	% within Do	0.5933	0.42	0.5133	0.8068	0.9835	1.0401	0.726164
14	Count	213	138	136	222	269	308	1286
	Expected Count	214.35	214.35	214.35	214.31	214.32	214.32	1286
	% within BL	16.563	10.731	10.575	17.263	20.918	23.95	100
	% within Do	0.71	0.46	0.4533	0.7401	0.8968	1.0268	0.714496
15	Count	208	183	182	226	256	268	1323
	Expected Count	220.52	220.52	220.52	220.48	220.49	220.49	1323
	% within BL	15.722	13.832	13.757	17.082	19.35	20.257	100
	% within Do	0.6933	0.61	0.6067	0.7535	0.8534	0.8935	0.735053
16	Count	202	124	138	167	242	295	1168
	Expected Count	194.68	194.68	194.68	194.65	194.65	194.65	1168
	% within BL	17.295	10.616	11.815	14.298	20.719	25.257	100
	% within Do	0.6733	0.4133	0.46	0.5568	0.8068	0.9835	0.648936
17	Count	203	153	161	202	276	295	1290
	Expected Count	215.02	215.02	215.02	214.98	214.99	214.99	1290
	% within BL	15.736	11.86	12.481	15.659	21.395	22.868	100
	% within Do	0.6767	0.51	0.5367	0.6734	0.9201	0.9835	0.716718
18	Count	276	177	212	272	326	368	1631
	Expected Count	271.85	271.85	271.85	271.81	271.82	271.82	1631
	% within BL	16.922	10.852	12.998	16.677	19.988	22.563	100
	% within Do	0.92	0.59	0.7067	0.9068	1.0868	1.2268	0.906177
19	Count	565	372	360	458	488	569	2812

			Expected Count	468.7	468.7	468.7	468.62	468.64	468.64	2812
			% within BL	20.092	13.229	12.802	16.287	17.354	20.235	100
			% within Do	1.8833	1.24	1.2	1.5269	1.6269	1.8969	1.562335
		20	Count	2919	1806	1992	2353	2851	3172	15093
			Expected Count	2515.7	2515.7	2515.7	2515.3	2515.3	2515.3	15093
			% within BL	19.34	11.966	13.198	15.59	18.89	21.016	100
			% within Do	9.73	6.02	6.64	7.8446	9.5046	10.575	8.385606
		21	Count	8383	9362	11153	11100	10792	9884	60674
			Expected Count	10113	10113	10113	10111	10112	10112	60674
			% within BL	13.816	15.43	18.382	18.294	17.787	16.29	100
			% within Do	27.943	31.207	37.177	37.006	35.978	32.951	33.710221
	Total		Count	30000	30000	30000	29995	29996	29996	179987
			Expected Count	30000	30000	30000	29995	29996	29996	179987
			% within BL	16.668	16.668	16.668	16.665	16.666	16.666	100
			% within Do	100	100	100	100	100	100	100
12000	BL	1	Count	13799	17392	15881	14404	13694	12935	88105
			Expected Count	14685	14685	14684	14684	14683	14684	88105
			% within BL	15.662	19.74	18.025	16.349	15.543	14.681	100
			% within Do	45.997	57.973	52.938	48.015	45.653	43.12	48.9494
		2	Count	1023	725	596	610	758	1019	4731
			Expected Count	788.54	788.54	788.51	788.51	788.43	788.48	4731
			% within BL	21.623	15.324	12.598	12.894	16.022	21.539	100
			% within Do	3.41	2.4167	1.9867	2.0334	2.527	3.3969	2.62845
		3	Count	634	458	361	398	427	579	2857
			Expected Count	476.19	476.19	476.17	476.17	476.12	476.16	2857
			% within BL	22.191	16.031	12.636	13.931	14.946	20.266	100
			% within Do	2.1133	1.5267	1.2034	1.3267	1.4235	1.9301	1.587293
		4	Count	331	214	206	203	279	415	1648
			Expected Count	274.68	274.68	274.67	274.67	274.64	274.66	1648
			% within BL	20.085	12.985	12.5	12.318	16.93	25.182	100
			% within Do	1.1033	0.7133	0.6867	0.6767	0.9301	1.3834	0.915596
		5	Count	392	278	245	291	374	471	2051
			Expected Count	341.85	341.85	341.84	341.84	341.8	341.83	2051
			% within BL	19.113	13.554	11.945	14.188	18.235	22.964	100
			% within Do	1.3067	0.9267	0.8167	0.97	1.2468	1.5701	1.139495
		6	Count	350	222	189	252	329	413	1755
			Expected Count	292.51	292.51	292.5	292.5	292.47	292.49	1755
			% within BL	19.943	12.65	10.769	14.359	18.746	23.533	100
			% within Do	1.1667	0.74	0.63	0.84	1.0968	1.3768	0.975043
		7	Count	265	146	180	247	306	401	1545
			Expected Count	257.51	257.51	257.5	257.5	257.48	257.49	1545
			% within BL	17.152	9.4498	11.65	15.987	19.806	25.955	100
			% within Do	0.8833	0.4867	0.6	0.8234	1.0201	1.3368	0.858371
		8	Count	275	177	173	207	269	295	1396
			Expected Count	232.68	232.68	232.67	232.67	232.65	232.66	1396

	% within BL	19.699	12.679	12.393	14.828	19.269	21.132	100
	% within Do	0.9167	0.59	0.5767	0.69	0.8968	0.9834	0.77559
9	Count	214	132	142	185	222	262	1157
	Expected Count	192.84	192.84	192.84	192.84	192.82	192.83	1157
	% within BL	18.496	11.409	12.273	15.99	19.188	22.645	100
	% within Do	0.7133	0.44	0.4733	0.6167	0.7401	0.8734	0.642806
10	Count	294	183	207	246	277	355	1562
	Expected Count	260.34	260.34	260.34	260.34	260.31	260.33	1562
	% within BL	18.822	11.716	13.252	15.749	17.734	22.727	100
	% within Do	0.98	0.61	0.69	0.82	0.9235	1.1834	0.867816
11	Count	256	193	174	204	242	328	1397
	Expected Count	232.84	232.84	232.84	232.84	232.81	232.83	1397
	% within BL	18.325	13.815	12.455	14.603	17.323	23.479	100
	% within Do	0.8533	0.6433	0.58	0.68	0.8068	1.0934	0.776146
12	Count	246	137	133	182	278	288	1264
	Expected Count	210.68	210.68	210.67	210.67	210.65	210.66	1264
	% within BL	19.462	10.839	10.522	14.399	21.994	22.785	100
	% within Do	0.82	0.4567	0.4433	0.6067	0.9268	0.9601	0.702253
13	Count	189	131	140	205	250	285	1200
	Expected Count	200.01	200.01	200	200	199.98	200	1200
	% within BL	15.75	10.917	11.667	17.083	20.833	23.75	100
	% within Do	0.63	0.4367	0.4667	0.6834	0.8334	0.9501	0.666696
14	Count	225	142	129	187	229	262	1174
	Expected Count	195.68	195.68	195.67	195.67	195.65	195.66	1174
	% within BL	19.165	12.095	10.988	15.928	19.506	22.317	100
	% within Do	0.75	0.4733	0.43	0.6234	0.7634	0.8734	0.652251
15	Count	223	187	192	226	228	234	1290
	Expected Count	215.01	215.01	215	215	214.98	215	1290
	% within BL	17.287	14.496	14.884	17.519	17.674	18.14	100
	% within Do	0.7433	0.6233	0.64	0.7534	0.7601	0.7801	0.716699
16	Count	211	129	132	143	199	247	1061
	Expected Count	176.84	176.84	176.84	176.84	176.82	176.83	1061
	% within BL	19.887	12.158	12.441	13.478	18.756	23.28	100
	% within Do	0.7033	0.43	0.44	0.4767	0.6634	0.8234	0.589471
17	Count	213	146	157	187	243	249	1195
	Expected Count	199.18	199.18	199.17	199.17	199.15	199.16	1195
	% within BL	17.824	12.218	13.138	15.649	20.335	20.837	100
	% within Do	0.71	0.4867	0.5234	0.6234	0.8101	0.8301	0.663918
18	Count	288	178	198	243	274	315	1496

			Expected Count	249.34	249.34	249.34	249.34	249.31	249.33	1496
			% within BL	19.251	11.898	13.235	16.243	18.316	21.056	100
			% within Do	0.96	0.5933	0.66	0.81	0.9135	1.0501	0.831148
		19	Count	580	379	356	408	416	475	2614
			Expected Count	435.69	435.69	435.67	435.67	435.63	435.66	2614
			% within BL	22.188	14.499	13.619	15.608	15.914	18.171	100
			% within Do	1.9333	1.2633	1.1867	1.36	1.3869	1.5834	1.452287
		20	Count	2898	1688	1746	1952	2278	2459	13021
			Expected Count	2170.3	2170.3	2170.2	2170.2	2170	2170.1	13021
			% within BL	22.256	12.964	13.409	14.991	17.495	18.885	100
			% within Do	9.66	5.6267	5.8202	6.5069	7.5943	8.1972	7.23421
		21	Count	7094	6763	8462	9019	8424	7711	47473
			Expected Count	7912.5	7912.5	7912.3	7912.3	7911.5	7912	47473
			% within BL	14.943	14.246	17.825	18.998	17.745	16.243	100
			% within Do	23.647	22.543	28.208	30.064	28.084	25.705	26.37506
	Total		Count	30000	30000	29999	29999	29996	29998	179992
			Expected Count	30000	30000	29999	29999	29996	29998	179992
			% within BL	16.667	16.667	16.667	16.667	16.665	16.666	100
			% within Do	100	100	100	100	100	100	100
14000	BL	1	Count	14429	18980	18598	16751	15760	14086	98604
			Expected Count	16620	16620	16619	16617	16618	15509	98604
			% within BL	14.633	19.249	18.861	16.988	15.983	14.285	100
			% within Do	48.097	63.267	62	55.846	52.54	50.316	55.4008
		2	Count	1088	791	641	625	686	860	4691
			Expected Count	790.69	790.69	790.61	790.56	790.59	737.85	4691
			% within BL	23.193	16.862	13.664	13.323	14.624	18.333	100
			% within Do	3.6267	2.6367	2.1369	2.0837	2.287	3.072	2.635645
		3	Count	663	490	377	399	414	452	2795
			Expected Count	471.11	471.11	471.07	471.03	471.05	439.63	2795
			% within BL	23.721	17.531	13.488	14.275	14.812	16.172	100
			% within Do	2.21	1.6333	1.2568	1.3302	1.3802	1.6146	1.570375
		4	Count	349	207	204	194	267	339	1560
			Expected Count	262.95	262.95	262.92	262.9	262.91	245.37	1560
			% within BL	22.372	13.269	13.077	12.436	17.115	21.731	100
			% within Do	1.1633	0.69	0.6801	0.6468	0.8901	1.2109	0.876488
		5	Count	424	293	266	280	361	385	2009
			Expected Count	338.63	338.63	338.59	338.57	338.58	316	2009
			% within BL	21.105	14.584	13.24	13.937	17.969	19.164	100
			% within Do	1.4133	0.9767	0.8868	0.9335	1.2035	1.3752	1.128759
		6	Count	366	230	183	243	290	333	1645
			Expected Count	277.27	277.27	277.25	277.23	277.24	258.74	1645
			% within BL	22.249	13.982	11.125	14.772	17.629	20.243	100
			% within Do	1.22	0.7667	0.6101	0.8101	0.9668	1.1895	0.924246
		7	Count	278	161	180	242	282	330	1473

	Expected Count	248.28	248.28	248.26	248.24	248.25	231.69	1473
	% within BL	18.873	10.93	12.22	16.429	19.145	22.403	100
	% within Do	0.9267	0.5367	0.6001	0.8068	0.9401	1.1788	0.827607
8	Count	301	212	161	197	242	252	1365
	Expected Count	230.08	230.08	230.06	230.04	230.05	214.7	1365
	% within BL	22.051	15.531	11.795	14.432	17.729	18.462	100
	% within Do	1.0033	0.7067	0.5367	0.6568	0.8068	0.9002	0.766927
9	Count	226	125	140	174	206	217	1088
	Expected Count	183.39	183.39	183.37	183.36	183.36	171.13	1088
	% within BL	20.772	11.489	12.868	15.993	18.934	19.945	100
	% within Do	0.7533	0.4167	0.4667	0.5801	0.6868	0.7751	0.611294
10	Count	315	208	195	240	264	295	1517
	Expected Count	255.7	255.7	255.67	255.66	255.66	238.61	1517
	% within BL	20.765	13.711	12.854	15.821	17.403	19.446	100
	% within Do	1.05	0.6933	0.6501	0.8001	0.8801	1.0538	0.852329
11	Count	271	197	169	202	224	259	1322
	Expected Count	222.83	222.83	222.81	222.79	222.8	207.94	1322
	% within BL	20.499	14.902	12.784	15.28	16.944	19.592	100
	% within Do	0.9033	0.6567	0.5634	0.6734	0.7468	0.9252	0.742768
12	Count	254	135	134	159	237	233	1152
	Expected Count	194.18	194.18	194.16	194.14	194.15	181.2	1152
	% within BL	22.049	11.719	11.632	13.802	20.573	20.226	100
	% within Do	0.8467	0.45	0.4467	0.5301	0.7901	0.8323	0.647253
13	Count	195	141	133	185	233	230	1117
	Expected Count	188.28	188.28	188.26	188.25	188.25	175.69	1117
	% within BL	17.457	12.623	11.907	16.562	20.859	20.591	100
	% within Do	0.65	0.47	0.4434	0.6168	0.7768	0.8216	0.627588
14	Count	232	146	125	167	199	217	1086
	Expected Count	183.05	183.05	183.03	183.02	183.03	170.82	1086
	% within BL	21.363	13.444	11.51	15.378	18.324	19.982	100
	% within Do	0.7733	0.4867	0.4167	0.5568	0.6634	0.7751	0.610171
15	Count	228	186	185	217	206	200	1222
	Expected Count	205.97	205.97	205.95	205.94	205.95	192.21	1222
	% within BL	18.658	15.221	15.139	17.758	16.858	16.367	100
	% within Do	0.76	0.62	0.6167	0.7235	0.6868	0.7144	0.686582
16	Count	216	118	123	133	167	201	958
	Expected Count	161.48	161.48	161.46	161.45	161.45	150.68	958
	% within BL	22.547	12.317	12.839	13.883	17.432	20.981	100
	% within Do	0.72	0.3933	0.41	0.4434	0.5567	0.718	0.538254
1	Count	218	142	159	177	206	195	1097

7	Expected Count	184.91	184.91	184.89	184.87	184.88	172.55	1097
	% within BL	19.872	12.944	14.494	16.135	18.778	17.776	100
	% within Do	0.7267	0.4733	0.5301	0.5901	0.6868	0.6966	0.616351
18	Count	291	180	188	240	254	260	1413
	Expected Count	238.17	238.17	238.14	238.13	238.14	222.25	1413
	% within BL	20.594	12.739	13.305	16.985	17.976	18.401	100
	% within Do	0.97	0.6	0.6267	0.8001	0.8468	0.9287	0.793896
19	Count	590	375	349	382	377	382	2455
	Expected Count	413.8	413.8	413.76	413.73	413.75	386.15	2455
	% within BL	24.033	15.275	14.216	15.56	15.356	15.56	100
	% within Do	1.9667	1.25	1.1634	1.2735	1.2568	1.3645	1.379345
20	Count	2869	1568	1538	1657	1957	1969	11558
	Expected Count	1948.2	1948.2	1948	1947.8	1947.9	1818	11558
	% within BL	24.823	13.566	13.307	14.336	16.932	17.036	100
	% within Do	9.5633	5.2267	5.1272	5.5243	6.5242	7.0334	6.493879
21	Count	6197	5115	5949	7131	7164	6300	37856
	Expected Count	6380.8	6380.8	6380.2	6379.8	6380	5954.4	37856
	% within BL	16.37	13.512	15.715	18.837	18.924	16.642	100
	% within Do	20.657	17.05	19.832	23.774	23.883	22.504	21.26945
Total	Count	30000	30000	29997	29995	29996	27995	177983
	Expected Count	30000	30000	29997	29995	29996	27995	177983
	% within BL	16.856	16.856	16.854	16.853	16.853	15.729	100
	% within Do	100	100	100	100	100	100	100